

# Subjective Return Expectations

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## Abstract

I document that the return expectations of Wall Street analysts are contrarian and countercyclical, contrasting with existing evidence that return expectations among Main Street investors (CFOs, retail investors) appear exclusively extrapolative and positively correlated. I demonstrate that an expectation formation framework in which investors use imperfect predictors to minimize forecast errors can rationalize these facts. Estimating the framework using surveys, I find Wall Street and Main Street disagree on what fundamental news means for future returns while agreeing on a persistent fundamental process driving most variations of asset prices. These results support models featuring heterogeneous agents with persistent fundamental expectations.

**Keywords:** *Rational Expectation; Expected Return; Belief Formation; Excess Volatility; Wall Street Analyst; Main Street; Stock Market.*

**JEL Classification:** G12; G17; D84.

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An investor deciding whether to hold cash or invest in stocks forms their own (subjective) return expectation. This expectation drives their asset allocation decision and, in the aggregate, asset prices. Understanding investors' subjective return expectations should therefore be of first-order importance for studying asset prices. Yet, the literature exploring this research venue is surprisingly limited, arguably due to the rational expectation assumption adopted by many studies.<sup>1</sup> However, the seminal work of [Greenwood and Shleifer \(2014\)](#) shows investors' subjective return expectations can deviate significantly from the rational expectation benchmark, which motivates [Brunnermeier et al. \(2021\)](#) to call on researchers to do more work on subjective expectations.

This paper makes contributions along this line of inquiry. Specifically, I try to address the questions of who holds what kind of return expectations, and how investors form their return expectation in real time. The main contribution of this paper is to document contrarian and counter-cyclical subjective return expectations of Wall Street analysts (sell-side and buy-side) and to propose an expectation formation framework to understand investors' *heterogeneous* return expectations—contrarian or extrapolative.<sup>2</sup> Additionally, I also provide new empirical moments to distinguish asset pricing theories based on data on subjective expectations. At a high level, the results in this paper expand our understanding about subjective return expectations: they do not have to be exclusively extrapolative as in [Greenwood and Shleifer \(2014\)](#), nor do they need to be irrational— they could instead be a result of investors' optimally learning from past information when facing the formidable task of forecasting future returns.

More specifically, my findings are threefold. First, I find that the subjective return expectations of Wall Street analysts (buy side and sell side) appear strongly contrarian and countercyclical. Furthermore, they are strongly negatively correlated with the existing return expectations documented in the literature,<sup>3</sup> which shows that subjective return expectations of Main Street investors (e.g., retail investors, pension fund managers, and CFOs) seem exclusively extrapolative and are positively correlated with each other. This Wall Street vs. Main Street market structure

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<sup>1</sup>Assuming rational expectations, direct data on investors' surveys are redundant, since investors would always hold rational expectations. There is also a separate literature that studies the objective, or rational expected returns from the point of view of an econometrician, namely the return predictability literature. One of the main findings of the literature is that future returns are predictable in sample by fundamental ratios such as the dividend-price or earnings-price ratios. See [Cochrane \(2011\)](#); [R. S. Koijen and Van Nieuwerburgh \(2011\)](#) for reviews.

<sup>2</sup>More explicitly, extrapolative return expectations means investors return expectations are positively correlated with past realized returns.

<sup>3</sup>See for example, [Andonov and Rauh \(2020\)](#); [Greenwood and Shleifer \(2014\)](#); [Vissing-Jorgensen \(2003\)](#).

paints a more complete picture of who holds what expectations in the real world.

Second, through the expectation formation framework proposed here, I find that neither Wall Street's nor Main Street's expectation needs to be irrational, as commonly interpreted in the literature. Instead, different return expectations could all be a result of investors optimally learning from past information, including past returns and fundamental-price ratios. Subjective return expectations may differ because past information is subject to interpretations due to a well-known parameter identification problem and because investors have different preferences and personal experiences.

Third, by estimating the expectation formation framework using surveys, I find that CFOs, pension funds, and sell-side analysts all believe economic fundamentals are persistent and drive asset price variations, despite having differences in interpreting the relationship of cash flow news and future returns. These results support asset pricing models featuring heterogeneous agents with persistent expectations about fundamentals.

In what follows, I explain briefly how I arrive at these three findings. First, to arrive at the finding that the subjective return expectations of Wall Street analysts are contrarian and countercyclical, I start by assembling data sets on return expectations of Wall Street analysts, both sell side and buy side. The sell-side analysts' return expectations are based on a comprehensive survey of analysts' price targets over 20 years, which includes about 2700 analysts on average at a point in time. The buy-side analysts' return expectations are hand collected from a well-known asset allocation fund which publishes return expectations for U.S. equities. Wall Street analysts' return expectations should be important in shaping asset prices, yet the literature has largely overlooked how they vary over time, especially on the aggregate level. Furthermore, the literature has also largely failed to address how Wall Street analysts' return expectations correlate with other documented return expectations, such as those of CFOs, retail investors, or pension fund managers.

Analysis of these data shows that Wall Street analysts' return expectations are volatile, contrarian, and countercyclical. The aggregate return expectations of sell-side analysts have an annual volatility of 13%, and they decrease by 0.16% for each one percent increase in past six month returns; the correlation between the buy-side analysts' return expectations and Shiller's CAPE ratio is -0.81. Furthermore, there is a clear Wall Street and Main Street structure among

these return expectations. The correlations between sell-side and buy-side analysts' expectations and those of consumers are -0.68 and -0.69, respectively, while the correlation between sell-side and buy-side analysts is 0.41.

Second, to interpret these findings, I propose a framework of return expectation formation in which investors (i) use imperfect predictors, (ii) *rationaly* form and update their beliefs, and (iii) start from different subjective priors. Compared to the common interpretation of investors' subjective return expectations in the literature—that investors are irrational—in this framework, investors are bounded rational. I briefly explain how this seemingly realistic and rational framework accounts for the heterogeneous return expectations observed in the data as follows.

The first element of the framework—the imperfect predictors—refers to the commonly used fundamental-price ratios, such as dividend-price or earnings-price ratios. These are indeed noisy predictors of future returns, as the literature has shown. Theoretically, the present-value relationship implies that these ratios are driven by two latent economic forces, namely, expected return and expected future cash flow growth; empirically, their out-of-sample predictive performance has been shown to be unstable.

The second element—investors rationally forming and updating their beliefs—means that investors are aware of the predictors' imperfections (they understand the present-value relation) and use Kalman filters to update their expectations. That the investors minimize their prospective forecast errors given the information available to them, including the predictors and past realized returns, shows they are acting rationally. But given that all investors optimize based on the same information, why do they persistently disagree with each other?

The disagreement persists because investors forming return expectations face a fundamental parameter identification problem that is well known to researchers in the return predictability literature.<sup>4</sup> Indeed, different interpretations of the world (asset pricing models) can be justified by the same set of observable data, hence investors can agree to disagree. Having observed an increase in a valuation ratio, such as the dividend yield, one investor could interpret it as the market's increased discount rate, while the other can freely believe it comes from decreased future fundamental growth. Both investors' beliefs are reasonable based on their own prior beliefs.

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<sup>4</sup>Papers discussing this identification issue include [Cochrane \(2008\)](#); [R. S. Kojen and Van Nieuwerburgh \(2011\)](#); [Pástor and Stambaugh \(2009\)](#); [Rytchkov \(2012\)](#).

The differing priors at the root of the disagreement are the third element of the framework. Investors differ in their own personal experiences and their views of the world. Indeed, literature on expectation formation finds personal experiences lead to differential expectations. Furthermore, investors may have different views about how aggregate asset prices evolve and thus form different return expectations.

Indeed, differences among asset pricing models reflect these different priors. For example, in [Campbell and Cochrane \(1999\)](#), the rational representative agent would interpret the increase in dividend yield as a sign of higher future returns because they believe the cash flow process is independently and identically distributed (i.i.d.) and only the discount rate is persistent and predictable. On the other hand, in models featuring persistent long-run risks, agents hold objective ([Bansal and Yaron \(2004\)](#)) or subjective beliefs ([Collin-Dufresne, Johannes, and Lochstoer \(2016\)](#)) that the cash flow process contains a persistent component. In these models, the rational representative agents may well interpret the increase in dividend yield as a sign of lower expected returns, if they believe a higher expected future cash flow and expected returns are positively related.<sup>5</sup>

Therefore, the expectation formation framework proposed here can be used together with survey data to identify investors' prior beliefs and thus provides a new testing ground for different asset pricing models—the paper's third contribution. More specifically, I demonstrate that, in order to form a unique return expectation, investors need to impose prior beliefs on (i) the relative importance of cash flow news in driving valuations compared to discount rate news, and (ii) the correlation between cash flow news and expected return news. I further confirm through simulation analysis that moderate differences in the two prior beliefs can reproduce the heterogeneous return expectations observed in the data.<sup>6</sup>

I apply the framework to survey data. My main estimation results, presented in Section 3, are

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<sup>5</sup>In the models featuring persistent expected cash flows, the correlation between expected cash flow news and expected return news depends typically on the relative magnitude of the representative agent's inter-temporal elasticity of substitution (IES) and relative risk aversion. More specifically, when an agent has a high IES (e.g.,  $IES > 1$ ) relative to risk aversion, they prefer to consume less in light of higher expected growth, leading to a lower expected return. On the other hand, when their IES is small relative to risk aversion ( $IES < 1$ ), they prefer to consume more now (facing positive growth news), leading them to believe the risk premium is higher and positively related to expected growth news.

<sup>6</sup>Interestingly, academic researchers are still debating about the objective values of these parameters. The debate about the persistence parameter, or how persistent a cash flow shock actual is, centers around the predictability of cash flows, see [Bansal, Kiku, and Yaron \(2012\)](#) and [Beeler \(2012\)](#). For a discussion on the correlation between discount rate and cash flow shocks, see [Lochstoer and Tetlock \(2020\)](#).

as follows. First, both Wall Street and Main Street believe the expected cash flow process is more persistent than the discount rate process and, moreover, that the expected cash flow process is the main economic force driving asset price variations. Second, Wall Street believes that positive cash flow news leads to lower future returns, while Main Street believes the opposite. These results support models featuring heterogeneous agents with persistent expected fundamental processes.

Additionally, I explore channels that lead investors to have different sets of prior beliefs in the first place. The literature suggests personal experiences as a key variable driving investors' subjective beliefs, either through perceptions about economic variables ([Malmendier and Nagel \(2016\)](#)) or through risk appetite ([Malmendier and Nagel \(2011\)](#)). These two channels echo the two prior beliefs highlighted in the expectation formation framework proposed in this paper. Taking advantage of the breadth of the sell-side analysts' survey, I find that more experienced analysts are indeed more contrarian. This result further supports models featuring agents learning from experiences, such as [Collin-Dufresne, Johannes, and Lochstoer \(2017\)](#) and [Nagel and Xu \(2019\)](#).<sup>7</sup>

Admittedly, the current study has limitations. First, the surveys studied here, although comprehensive, do not cover all of the financial market participants. Expectations for a large portion of market participants are still missing. Second, the expectation formation framework relies on the assumption that the underlying cash flow and discount rate processes follow AR(1). Although this assumption is common in the finance literature, it precludes richer dynamics of these processes, including a term structure of expectation dynamics. Finally, there is a missing connection between subjective beliefs and asset prices, namely investors' holdings. Frictions may exist from expectations to investors' actual holdings, although this issue persists for all studies on subjective beliefs.

## Related Literature

This paper relates to a growing literature that studies investors' subjective expectations, specifically contributing to three strands of this literature.

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<sup>7</sup>In [Collin-Dufresne et al. \(2016\)](#), subjective expected consumption growth is dependent on the number of years an agent learns about the endowment process. In [Nagel and Xu \(2019\)](#), the subjective expected consumption growth is related to an agent's experienced payouts.

First, the paper contributes to literature that uses survey data to study empirical properties of subjective return expectations. [Vissing-Jorgensen \(2003\)](#) and [Greenwood and Shleifer \(2014\)](#) study surveys conducted for CFOs, retail investors, and consumers; more recently, [Andonov and Rauh \(2020\)](#) study return expectations of pension funds. [Wu \(2018\)](#) finds sell-side analysts' return expectations are positively correlated with future realized market excess returns and contemporaneous market volatility measures. This paper focuses on how buy-side and sell-side analysts return expectations are related to past realized returns and valuation ratios and compare their expectations with those of CFOs and retail investors, who exhibit extrapolative beliefs. Furthermore, this paper analyzes the dynamic behaviors of multiple survey-based expectations and provides a framework to understand why and how they would differ in their dynamics.

Second, the paper also contributes to literature that studies subjective return expectation formation. Authors have turned to learning, often via non-Bayesian updating schemes ([Adam, Marcet, & Beutel, 2017](#); [Bordalo, Gennaioli, & Shleifer, 2018](#); [Branch & Evans, 2010](#); [Nagel & Xu, 2019](#)), to generate the observed subjective return expectations that deviate from rational benchmarks. Additionally, other authors have identified personal experiences as an important driver for return expectations ([Kuchler & Zafar, 2019](#); [Malmendier & Nagel, 2011, 2016](#)). I build on the insights from the previous literature and take into account the new facts documented here — the contrarian and countercyclical expectations — when develop the model of expectation formation. As a result, the expectation formation framework differs from those in the previous literature. The heterogeneity in return expectations based on the framework proposed here is endogenous to the parameter identification problem inherent in the return forecasting problem in the presence of noisy predictors—a reality that Bayesian econometricians also have to face. Differences in personal experiences constitute a potential channel through which resulting return expectations might differ. [Kindermann, Le Blanc, Piazzesi, and Schneider \(2020\)](#) also consider an expectation framework based on the present-value relation, although in the German housing market. Different from this paper, the heterogeneity in return expectation in their framework is generated by different agents receiving different signals due to different experiences, whereas investors in this paper receive the same signal but make different inferences from it.

Finally, the paper contributes to the literature that jointly studies data on subjective expecta-

tion and asset prices (Adam et al., 2017; Barberis, Greenwood, Jin, & Shleifer, 2015; Bordalo et al., 2018; Collin-Dufresne et al., 2016; Hirshleifer, Li, & Yu, 2015; Nagel & Xu, 2019). This paper contributes to this literature by proposing a new framework to distinguish asset pricing theories using subjective return expectation data, whereas the literature tries to propose new asset pricing models that could jointly match subjective return expectations and asset prices. De la O and Myers (2020) consider sell-side analysts' earnings estimates and CFOs' return expectations to test asset pricing models. Their approach uses the decomposition of price-earnings and price-dividend ratios. My paper considers return expectations of sell-side and buy-side analysts, which are contrarian and countercyclical. Furthermore, the tests conducted in this paper make use of the newly proposed expectation formation framework to uncover a richer set of moments, such as the correlation between (subjective) discount rate and cash flow news, in addition to the variance decomposition of price-fundamental ratios. Thus, I can make finer distinctions between asset pricing models.

The rest of the paper is organized as follows. I document new facts about subjective return expectations in Section 1; I present and demonstrate the expectation formation framework in Section 2; and I estimate prior beliefs governing the expectation formation process in Section 3 before concluding in Section 4.

## **1 Heterogeneous Return Expectations and the Contrarian Wall Street Analysts**

In this section, I document new facts to extend our understanding about subjective return expectations. First, I assemble a comprehensive set of return expectations and demonstrate that the market structure of return expectations forms clusters within Wall Street (sell side, buy side) and Main Street (CFOs, consumers). Expectations are negatively correlated between the two clusters, and Wall Street expectations are more in line with measures of objective return expectations. Second, I zoom in on sell-side analysts and establish that they have contrarian return expectations on the market, firm, and analyst levels.

## 1.1 Data Sources and Measuring Wall Street Analysts Return Expectations

Table 1 summarizes the data sources. While subjective return expectations of other groups have been studied in the literature,<sup>8</sup> the current paper documents new facts about buy-side and sell-side analyst return expectations, which I briefly describe in this subsection. More details about data sources are provided in Appendix B.

The buy-side return expectation is from the asset management firm Grantham, Mayo & Van Otterloo Company, LLC (GMO), which publishes a seven-year asset class forecast each quarter on their website.<sup>9</sup> The reason to use GMO's return expectations is twofold. First, seldom do any buy-side firms publish their return expectations, and GMO is the only source providing a long-term historical account of return forecasts back to the second quarter of 2000. Second, GMO runs a large asset allocation fund for which return expectations are important. Tower (2010) documents that GMO's return expectations actually predict the returns in Vanguard mutual funds with different asset classes and styles. GMO's expected returns for equities have been available on their website since 2017. For pre-2017 data, I hand collected the data from the internet.

Aggregate (S&P 500-level and firm-level) sell-side analysts' return expectations are constructed using individual analyst price targets. The expected returns are computed by dividing the individual analyst's price targets by the daily closing price on the day the estimates were issued, and then subtracting 1,<sup>10</sup> or

$$\mu_{i,f,d}^A = \frac{P_{i,f,d}^{A,12}}{P_{f,d}} - 1$$

where  $P_{i,f,d}^{A,12}$  is the price target of analyst  $i$  for firm  $f$ , issued at day  $d$ , and  $P_{f,d}$  is the closing price of the firm  $f$ . The superscript "A" denotes the 12-month ahead estimates.<sup>11</sup>

Firm-level return expectations, which are simple averages of analyst-level return expectations and market-level return expectations, are market-cap weighted firm-level return expectations. Details of these expectations' construction are found in Appendix A.1.<sup>12</sup>

The data set of sell-side analysts' price targets has comprehensive coverage, including, on

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<sup>8</sup>Examples include Adam, Matveev, and Nagel (2021); Ben-David, Graham, and Harvey (2013); Greenwood and Shleifer (2014); Vissing-Jorgensen (2003).

<sup>9</sup><https://www.gmo.com/americas/research-library/>

<sup>10</sup>The same formula is used in Brav and Lehavy (2003) and Da and Schaumburg (2011)

<sup>11</sup>12-month ahead estimates are the most commonly issued horizon.

<sup>12</sup>Notice this methodology ensures there is no mechanical relation between mean estimated expected returns and the level of prices. On each issuing date, the analyst has the freedom to pick their own price target since they observe the prices.

Table 1: Data Sources for Subjective and Objective Return Expectations

This table summarizes the data sources used in the analysis in this paper. The sample period is 2002-01-01 to 2018-12-31, for which all data is available. Appendix B provides more details about the data sources.

| Measures of Subjective Return Expectation |                                                                  |
|-------------------------------------------|------------------------------------------------------------------|
| Who                                       | Source                                                           |
| Sell-Side Analysts                        | I/B/E/S detailed unadjusted price targets                        |
| Buy-Side Analysts                         | Grantham, Mayo & Van Otterloo (GMO) 7-year Asset Class Forecasts |
| Institutional Investors (Pension)         | Shiller Survey/Yale University                                   |
| Survey of Professional Forecasters        | Federal Reserve                                                  |
| CFOs                                      | Duke University CFO Global Business Outlook                      |
| Retail Investors                          | Shiller Survey/Yale University                                   |
| Consumers                                 | University of Michigan Consumer Surveys                          |
| Proxies for Objective Return Expectations |                                                                  |
| Proxy                                     | Source                                                           |
| S&P 500 Price-Dividend Ratios             | GlobalX and CRSP                                                 |
| PE ratio                                  | Prof. Robert Shiller's Website                                   |
| Consumption-Wealth Ratio                  | Prof. Martin Lettau's Website                                    |

average, forecasts from about 2700 analysts from 236 brokerage firms at any point in time. I detail the coverage and the summary statistics in Internet Appendix I.1.

## 1.2 Heterogeneous Return Expectations: Wall Street vs Main Street

Table 2a shows correlations among surveys of different parties. The correlation matrix clearly displays a two-cluster structure: Wall Street vs. Main Street. Sell-side analysts, buy-side analysts, and professional forecasters at the Federal Reserve Bank form a Wall Street cluster, in which return expectations are positively correlated with one another. On the other hand, retail investors, CFOs, and consumers form another cluster, which is conventionally thought of as Main Street investors. The latter cluster provides results consistent with Greenwood and Shleifer (2014). On the other hand, correlations of return expectations between these two clusters are negative,

with consumers and buy-side analysts having a -69% correlation at one extreme.

The last row of Table 2a reports how different subjective return expectations react to past six-month returns. Wall Street appears to have a contrarian view, while Main Street extrapolates. Although the extrapolative expectations have been documented previously, sell-side analysts' contrarian expectations have not been documented in the literature to the best of my knowledge. To show these results are robust, I provide a detailed analysis in Section 1.3.

Panel 2b shows correlations between subjective return expectations and proxies for objective return expectations considered in the literature. Wall Street analysts' expectations are negatively (positively) correlated with price-fundamental ratios (Consumption-Wealth Ratio), which means they are countercyclical. In particular, for a commonly used objective return expectation (Cochrane (2011)), "ER.Rational", the Wall Street and Main Street return expectations are positively and negatively correlated with high magnitude, respectively.

Different return expectations within each cluster also load differently on past returns and fundamental-price ratios as well as the risk-free rate. For example, buy-side analysts are more related to objective return expectations and are less driven by past returns, while sell-side analysts are more influenced by past returns and the risk-free rate. Since fundamental-price ratios are much more persistent than past realized returns, sell-side analysts' return expectations should be more volatile than those of buy-side analysts. Similar patterns can be found between CFOs and consumers.

Table 2: Heterogeneous Subjective Return Expectations

Panel (a) shows correlations between surveys of different groups' return expectations and their reaction to past six-month returns. Panel (b) shows correlations between subjective return expectations and proxies for objective return expectations. *ER.analyst* is the value-weighted sell-side analyst return expectations for the S&P 500 index. *ER.buy.side* is from GMO's equity forecasts. *Prof.forecaster* is from the Federal Reserve Survey of Professional Forecasters 10-year stock return data. *Shiller.institutional* is the 12-month return expectations for the DJI from the Shiller Survey of investment managers. *Shiller.retail* is the 12-month return expectations for the DJI from the Shiller Survey of wealthy individuals. *ER.CFO* is the CFO year-over-year return expectations from the Duke University survey. *ER.consumer* represents one-year return expectations from the Michigan Survey of households. *past.6m.cum.ret* are six-month cumulative returns on the S&P 500 index. *ER.Rational* is the fitted value of regressing future 12-month returns on CAY and log(P/D) from 1970–2019. CAY is the consumption-wealth ratio from Professor Martin Lettau's website. *Log(P/D)* is the log price-dividend ratio of the S&P 500 index. CAPE is the Schiller's cyclically adjusted price/earnings ratio. *GS10* is the yield on the 10-year constant maturity treasury. Data are based on quarterly series. More details about these surveys are in Appendix B. \*  $p < 0.1$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$ .

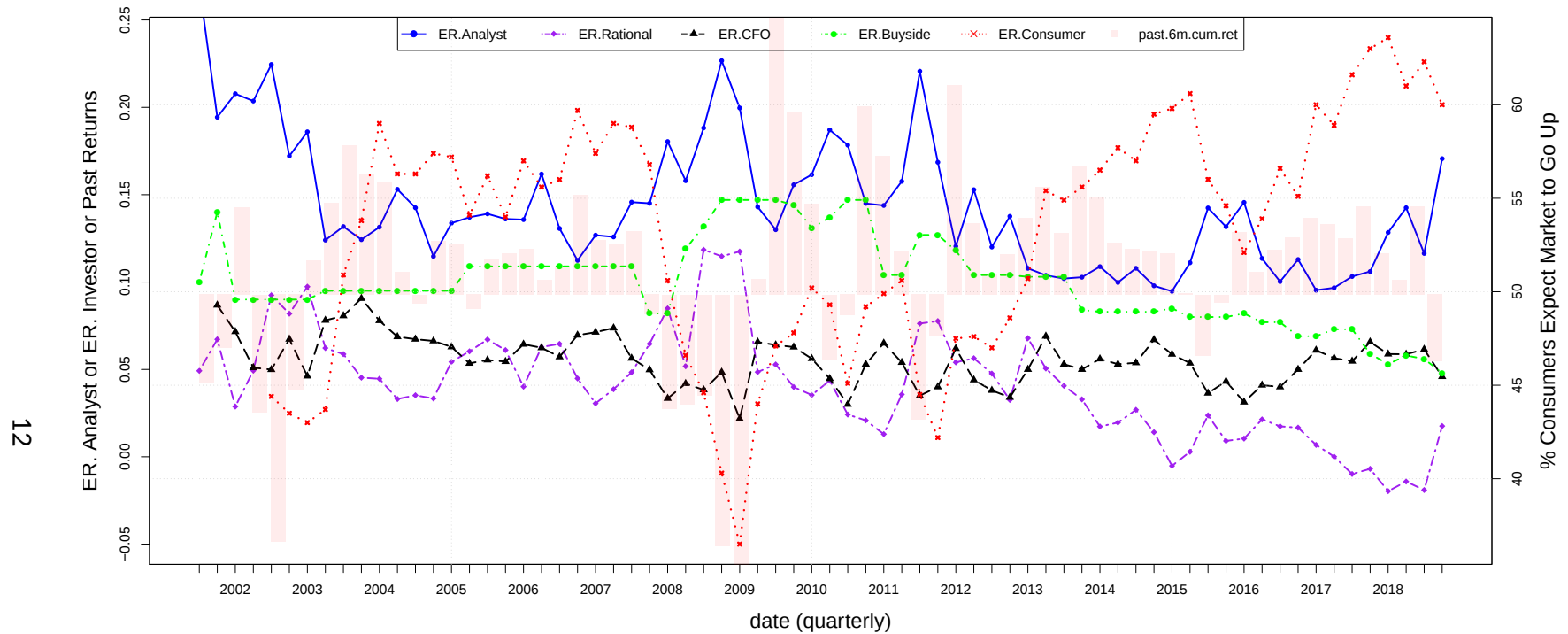
(a) Correlations Between Different Subjective Expectations

|                       | ER.analyst | ER.buy.side | Prof.forecaster | Shiller.institutional | Shiller.retail | ER.CFO  | ER.consumer |
|-----------------------|------------|-------------|-----------------|-----------------------|----------------|---------|-------------|
| ER.analyst            | 1.00       |             |                 |                       |                |         |             |
| ER.buy.side           | 0.41***    | 1.00        |                 |                       |                |         |             |
| Prof.forecaster       | 0.45***    | 0.53***     | 1.00            |                       |                |         |             |
| Shiller.institutional | 0.21*      | 0.26**      | 0.50***         | 1.00                  |                |         |             |
| Shiller.retail        | 0.00       | -0.20       | 0.30**          | 0.33**                | 1.00           |         |             |
| ER.CFO                | -0.27**    | -0.06       | 0.35***         | 0.20*                 | 0.68***        | 1.00    |             |
| ER.consumer           | -0.68***   | -0.69***    | -0.31**         | -0.37***              | 0.04           | 0.38*** | 1.00        |
| past.6m.cum.ret       | -0.65***   | -0.09       | -0.11           | 0.00                  | 0.08           | 0.48*** | 0.37***     |

(b) Correlation Between Analyst Expectations and Proxies of Objective Returns Expectations

|             | ER.analyst | ER.buy.side | ER.CFO  | ER.consumer |
|-------------|------------|-------------|---------|-------------|
| ER.analyst  | 1.00       |             |         |             |
| ER.buy.side | 0.43***    | 1.00        |         |             |
| ER.CFO      | -0.34***   | -0.13       | 1.00    |             |
| ER.consumer | -0.68***   | -0.69***    | 0.38*** | 1.00        |
| ER.rational | 0.61***    | 0.58***     | -0.20   | -0.73***    |
| CAY         | 0.54***    | 0.45***     | 0.08    | -0.55***    |
| log(P/D)    | -0.23*     | -0.36***    | 0.63*** | 0.52***     |
| CAPE        | -0.51***   | -0.81***    | 0.42*** | 0.90***     |
| GS10.pct    | 0.28**     | 0.19        | 0.51*** | 0.08        |

Figure 1: Market Structure for Return Expectations



Notes: “ER.Rational” is the fitted value of regressing future 12-month returns on CAY and  $\log(P/D)$  from 1970–2019. “past.6m.cum.ret” are the realized cumulative returns from the past six months, plotted as bars. “ER.consumer” are return expectations from a Michigan Survey of households asking what “percent chance” they thought that their investment would increase in value next year, which is measured on the right axis. More details about these surveys are in Appendix B.

Figure 1 visualizes these rich expectation dynamics. Indeed, sell-side analysts' return expectations are more volatile than those of buy-side analysts and CFOs. The persistent disagreement between consumers and sell-side analysts also stands out.

### 1.3 Contrarian Return Expectations of Sell-side Analysts

I demonstrate that the contrarian feature of sell-side analysts' return expectations is a robust finding. I run the time-series regression of aggregate analyst return expectations,  $\mu_{m,t}^A$ , on two-month lagged cumulative  $k$ -month past market returns,  $R_{m,t-2,t-k}$ , and other (lagged) control variables,  $X_{t-2}$  in the regression:<sup>13</sup>

$$\mu_{m,t}^A = a + bR_{m,t-2,t-k} + cX_{t-2} + e_t \quad (1)$$

A negative coefficient  $b$  would mean analysts expect the market to have a negative expected return following a past positive return, a contrarian expectation. Since the dependent variables are persistent, I use Newey-West standard errors with a 12-month lag to correct for autocorrelations. For control variables, I include the 10-year U.S. treasury yields and price-dividend ratios, as well as an analyst aggregate long-term growth measure to proxy for expected future earnings growth.

Table 4 shows the estimation results for regression 1. Table 3 shows the empirical distribution of the key variables in the regression to help interpret the magnitude of the coefficients. The coefficients on past returns are negative across all of the specifications and are significant both statistically and economically. In Column 1, for one standard deviation (percent) increase in the past six-month cumulative returns, the next month analyst return expectations decrease by 1.7% (0.16%), with a t-stat of 3.8. Since the monthly volatility of analyst return expectations is only 3.5%, the estimate indicates the economic magnitude of the contrarian effect is also large.

The contrarian effect does not only apply to short-term past realized returns. Columns 2 and 3 show that the past 36-month returns have almost the same predictive power as the six-month

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<sup>13</sup>The independent variables are lagged by two months when entering into the regression to prevent the estimates of  $b$  from being contaminated by stale analysts forecasts. When constructing individual analyst return expectations, the analyst price targets are at most two months old by construction. Therefore, lagging two months when running the predictive regression ensures that all future return expectations are out of sample. As an example, when using the past six-month cumulative returns at the end of June 2005 to predict an analyst's aggregate return expectations at the end of August 2005, the oldest analyst return expectation is constructed using price targets and stock prices in early July 2005.

Table 3: Summary Statistics: Monthly Aggregate Expectation Data (S&amp;P 500 firms)

This table displays summary statistics for key regression variables. *ER.analyst* are value-weighted sell-side analyst return expectations for the S&P 500 index. *tot.ret.1m* are one-month total returns on the S&P 500 index. *past.6m.cum.ret* are six-month cumulative returns on the S&P 500 index. *past.36m.cum.ret* are thirty-six-month cumulative returns on the S&P 500 index. *GS10* is the yield on the 10-year constant maturity treasury. *Log(P/D)* is log price-dividend ratio of the S&P 500 index. *LTG* is the value-weighted analyst long-term growth expectation for the S&P 500 index. *avg.nr.firms.ER.analyst* is the average monthly number of firms that have analyst return expectations in the S&P 500 index. More details about the construction of index level subjective expectations can be found in Section 1.1 and Appendix B.

| Statistic               | N   | Mean    | St. Dev. | Min    | Pctl(25) | Pctl(75) | Max   |
|-------------------------|-----|---------|----------|--------|----------|----------|-------|
| ER.analyst              | 208 | 0.142   | 0.037    | 0.087  | 0.115    | 0.160    | 0.270 |
| tot.ret.1m              | 208 | 0.006   | 0.041    | -0.168 | -0.015   | 0.031    | 0.109 |
| past.6m.cum.ret         | 208 | 0.031   | 0.111    | -0.427 | -0.010   | 0.089    | 0.388 |
| past.36m.cum.ret        | 208 | 0.156   | 0.302    | -0.434 | -0.108   | 0.371    | 0.858 |
| GS10                    | 208 | 0.033   | 0.011    | 0.015  | 0.023    | 0.042    | 0.053 |
| log(P/D)                | 208 | 3.946   | 0.152    | 3.289  | 3.875    | 4.031    | 4.305 |
| LTG                     | 208 | 0.115   | 0.014    | 0.081  | 0.105    | 0.122    | 0.153 |
| avg.nr.firms.ER.analyst | 208 | 490.178 | 7.076    | 459    | 486      | 495      | 500   |

cumulative returns. In fact, one standard deviation increase in past 36-month returns decreases the future analyst return expectation by 1.8%, in addition to that of the past six-month returns. Furthermore, the past six-month cumulative returns and the 36-month cumulative returns together explain up to 41% of the time variation in monthly analyst return expectations. This high R-squared further demonstrates the economic magnitude of the contrarian effect. Results from Columns 2 and 3 also raise the question of which horizon of past returns matter most to analysts' future return expectations. I investigate this question in Internet Appendix I.3.

The contrarian results are hardly affected when including the other control variables, as shown in Column 4. The 10-year treasury yield is the only variable with a (marginal) significance in predicting analysts' return expectations. To understand the magnitude of the coefficient on the treasury yield, consider as a benchmark that analysts believe the risk-free rate is a constant and that the risk-free rate is a part of the expected future return. In this case, the coefficient should be 1. Therefore, an estimated coefficient of 0.843 means analysts expect a degree of persistence in the risk-free rate process.

The contrarian effect is neither a result of aggregation nor staleness of analyst forecasts. Internet Appendix I.3 and Appendix C.2 show the contrarian results hold at the firm level and the analyst level, respectively. Remarkably, the magnitude of the contrarian effects are similar at each level. Furthermore, the results on the analyst level are based on the analysts' first-ever fore-

Table 4: Aggregate Analyst Return Expectations and Past Returns

The table reports coefficient estimates of

$$\mu_{m,t}^A = a + bR_{m,t-2,t-k} + cX_{t-2} + e_t$$

$\mu_{m,t}^A$  are aggregate analyst return expectations;  $R_{m,t-2,t-6}$  (*past.6m.cum.ret*) and  $R_{m,t-2,t-36}$  (*past.36m.cum.ret*) are six-month and 36-month past cumulative returns, respectively. *GS10* is the yield on the 10-year constant maturity treasury. *Log(P/D)* is the log price-dividend ratio of the S&P 500 index. *Analyst LTG Estimate* is the value-weighted analyst long-term growth expectation for the S&P 500 index. Variables are lagged two months before entering the regressions. Sample period: 2002-03-01 to 2018-12-31, a total of 202 months. \*p<0.1; \*\*p<0.05; \*\*\*p<0.01. Standard errors (in brackets) are Newey-West SEs with 12-month lag.

|                             | Dependent variable:                   |                         |                         |                         |
|-----------------------------|---------------------------------------|-------------------------|-------------------------|-------------------------|
|                             | Aggregate Analyst Return Expectations |                         |                         |                         |
|                             | (1)                                   | (2)                     | (3)                     | (4)                     |
| <i>past.6m.cum.ret</i>      | -0.163***<br>(0.043)                  |                         | -0.125***<br>(0.033)    | -0.115***<br>(0.038)    |
| <i>past.36m.cum.ret</i>     |                                       | -0.062***<br>(0.022)    | -0.049***<br>(0.015)    | -0.038**<br>(0.018)     |
| <i>GS10</i>                 |                                       |                         |                         | 0.867*<br>(0.497)       |
| <i>Log(P/D)</i>             |                                       |                         |                         | -0.015<br>(0.042)       |
| <i>Analyst LTG Estimate</i> |                                       |                         |                         | 0.382<br>(0.370)        |
| Constant                    | 0.146***<br>(0.006)                   | 0.151***<br>(0.009)     | 0.152***<br>(0.006)     | 0.135<br>(0.129)        |
| Observations                | 206                                   | 206                     | 206                     | 206                     |
| R <sup>2</sup>              | 0.266                                 | 0.284                   | 0.428                   | 0.516                   |
| Adjusted R <sup>2</sup>     | 0.263                                 | 0.281                   | 0.422                   | 0.504                   |
| Residual Std. Error         | 0.030 (df = 204)                      | 0.030 (df = 204)        | 0.027 (df = 203)        | 0.025 (df = 200)        |
| F Statistic                 | 74.001*** (df = 1; 204)               | 81.031*** (df = 1; 204) | 75.862*** (df = 2; 203) | 42.706*** (df = 5; 200) |

casts, eliminating concerns that the contrarian effect is simply a result of stale analyst forecasts. Internet Appendix I.2 provides a detailed analysis on the timing and frequencies of analysts' price target forecasts.

## 2 A Framework for Subjective Return Expectation Formation

I propose an expectation formation framework to understand the subjective return expectation dynamics observed in the data. I start by describing the environment faced by investors in this framework. I present the subjective return expectation dynamics in this environment for an investor who minimizes their prospective forecast errors through Kalman filtering, and show through simulation that the framework can generate the rich patterns documented in Section 1. Subsequently, I use a simplified system (where investors only consider dividend yield and returns) to demonstrate the parameter identification problem faced by investors. Finally, I identify the

prior beliefs investors need to impose to arrive at various unique expectations.

## 2.1 The Environment

There are three types of shocks,  $\epsilon_d$ ,  $\epsilon_g$ , and  $\epsilon_\mu$ , which represent news about current dividends, expected future cash flow growth, and discount rates, respectively. These shocks follow a multivariate normal distribution:

$$\begin{pmatrix} \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix} \sim N \left[ 0, \begin{pmatrix} \sigma_d^2 & \sigma_{\mu d} & \sigma_{gd} \\ \sigma_{\mu d} & \sigma_\mu^2 & \sigma_{\mu g} \\ \sigma_{gd} & \sigma_{\mu g} & \sigma_g^2 \end{pmatrix} \right] \quad (2)$$

Next quarter dividend growth,  $\Delta d_t$ , contains a potentially persistent component  $g_t$ :

$$\Delta d_{t+1} = g_t + \epsilon_{d,t+1} \quad (3)$$

$$g_{t+1} = E_g(1 - \phi) + \phi g_t + \epsilon_{g,t+1} \quad (4)$$

Let  $\mu_t = E_t(r_{t+1})$  be the discount rate process, which follows an autoregressive process:

$$\mu_{t+1} = (1 - \beta)E_r + \beta\mu_t + \epsilon_{\mu,t+1} \quad (5)$$

Additionally, there exists a vector of return predictors, such as price-earnings ratios, whose values are correlated with the three shocks. I denote those as  $x_t$ , and they follow

$$x_{t+1} = (I - A)E_x + Ax_t + \epsilon_{x,t+1} \quad (6)$$

$x_t$  can be used to predict future returns because of its correlations with  $\epsilon_{\mu,t+1}$  via

$$\begin{pmatrix} \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \\ \epsilon_{x,t+1} \end{pmatrix} \sim N \left[ 0, \begin{pmatrix} \sigma_d^2 & \sigma_{\mu d} & \sigma_{gd} & \sigma'_{dx} \\ \sigma_{\mu d} & \sigma_\mu^2 & \sigma_{\mu g} & \sigma'_{\mu x} \\ \sigma_{gd} & \sigma_{\mu g} & \sigma_g^2 & \sigma'_{gx} \\ \sigma_{dx} & \sigma_{\mu x} & \sigma_{gx} & \sigma_x^2 \end{pmatrix} \right] \quad (7)$$

This environment is consistent with the literature on return predictability in which researchers aim to construct proxies for objective expected returns (Van Binsbergen and Koijen (2010), Pastor and Stambaugh (2009)). Although simple, the setup encapsulates asset pricing models featuring persistent cash flow and/or discount rate processes.

## 2.2 Investors' Subjective Return Expectation Formation Process

Investors do not observe the news directly, nor do they know the exact parameter values governing the data generating the process described in (5), (6), and (7). Instead, they observe changes in predictors, such as fundamental ratios and past returns, and then estimate news and parameter values from available information. Based on their estimates, they update their subjective return expectations through Kalman filters, or:

$$\widetilde{ER}_{t|t} = \widetilde{ER}_{t|t-1} + \widetilde{m}_t * (r_t - \widetilde{ER}_{t|t-1}) + \widetilde{n}_t * \epsilon_{x,t} \quad (8)$$

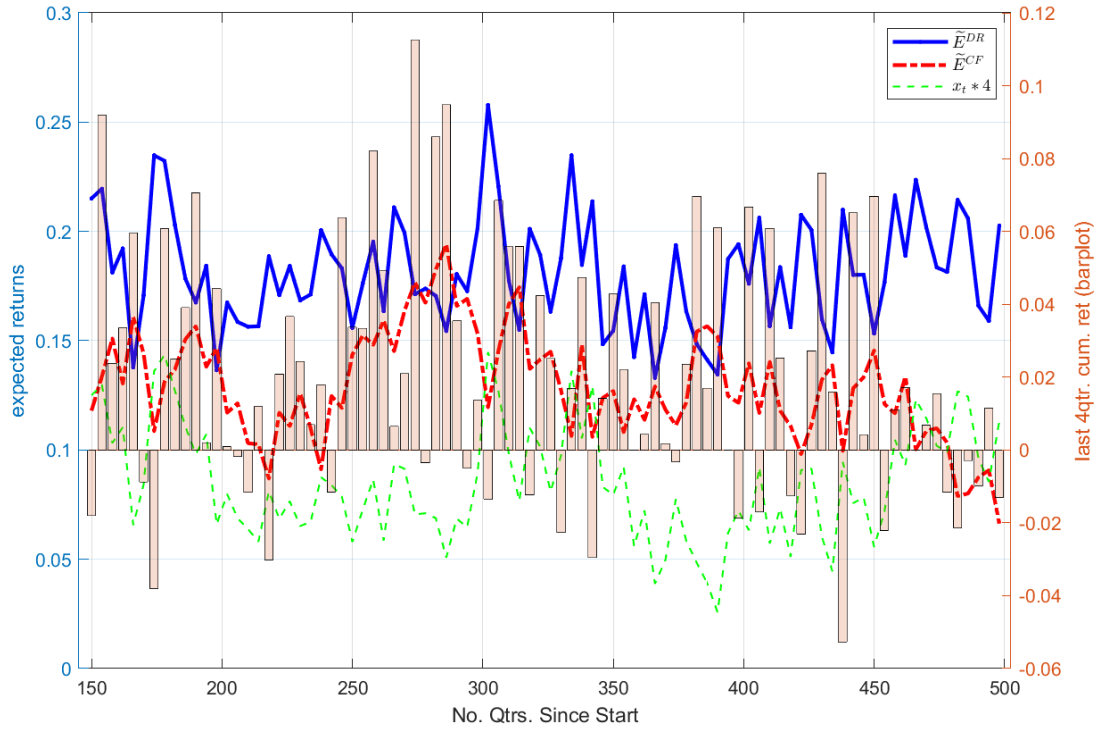
where  $\widetilde{ER}_{t|t} = \widetilde{E}(r_t|\mathcal{F}_t)$  is the subjective return expectation and  $\mathcal{F}_t$  denotes the agent's information set, which contains the values of all past predictors and past realized returns up to and including time  $t$ ;  $\sim$  means that the expectation depends on the agent's own subjective beliefs;  $\epsilon_{x,t}$  is the innovation in predictors defined in (6).  $\widetilde{m}_t$  and  $\widetilde{n}_t$  are functions of both parameters in the system (3) to (7) and values of realized returns and  $\epsilon_{x,t}$ . The time subscript in  $\widetilde{m}_t$  and  $\widetilde{n}_t$  captures the fact that agents learn and adjust their expectation formation process over time. See Appendix E for detailed derivation to arrive at Equation (8) and expressions for  $m_t$  and  $n_t$  as well as their steady-state values. The return expectation in (8) connects investors' subjective returns expectations to past realized returns and return predictors through the investors' optimization processes. Thus, this framework rationalizes the fact that observed return expectations from surveys are empirically related to past returns and fundamental-price ratios. Namely, from the investors' perspective, they are simply trying to project the most accurate return expectations based on the news in  $\epsilon_t = (\epsilon_{d,t}, \epsilon_{\mu,t}, \epsilon_{g,t}, \epsilon_{x,t})'$ .<sup>14</sup>

The simple expectation formation framework can generate the heterogeneous return expect-

<sup>14</sup>In Appendix (D), I demonstrate in Figure (5) that it is advantageous for investors to use realized returns to forecast future returns, even though past returns as a standalone predictor do not forecast future returns in linear predictive regressions. In fact, in studies such as Van Binsbergen and Koijen (2010), past returns are used together with dividend yields to predict future returns.

tation dynamic similar to what we observed empirically in Figure 1. I confirm the framework's ability to generate heterogeneous return expectations through simulation in Figure 2. In this figure, two investors form different return expectations following Equation (8), even though they observe the same information (the same past realized returns and dividend yields), and both investors try to minimize forecast errors using the same algorithm. The key difference between the investors is that one believes in a model close to the habit formation model of [Campbell and Cochrane \(1999\)](#) ( $\widetilde{ER}_t^{DR}$ ), and the other believes in a model in the same spirit of [Bansal and Yaron \(2004\)](#) ( $\widetilde{ER}_t^{CF}$ ). Next, I explain how and why this framework can generate such return expectations.

Figure 2: Simulated Return Expectations vs. Predictors and Past Returns Based on the Proposed Expectation Formation Dynamics



**Note:** The simulated return expectations are based on Equation (8).  $\widetilde{ER}_t^{DR}$  denotes the (annualized) return expectation of an investor who believes asset prices are 97% driven by discount rate variations;  $\widetilde{ER}_t^{CF}$  denotes the (annualized) return expectation of an investor who believes asset prices are 99% driven by expected cash flow growth variations;  $x_t * 4$  are dividend yields times 4; orange bars represent cumulative past six months returns. The dividend yields and past returns are simulated based on moments calibrated to the historical data. More details of the simulations are in Appendix D.

## 2.3 Understanding the Return Expectation Formation Process: A Simple One-Predictor System

### 2.3.1 Why Do Subjective Return Expectations Differ?

The reason behind the persistent disagreement in subjective return expectations is that investors face a parameter identification problem when predictors for future returns are not perfect. Intu-

itively, when the magnitude of return predictability is small, as found in the literature,<sup>15</sup> different interpretations of the data may persist. The expectation formation framework provides an analytical base to understand why and how these differences persist. In this framework, imperfect predictors mean that agents believe that none of the shocks in predictors  $x_t$  and  $\epsilon_{x,t}$  have a correlation with  $\epsilon_{\mu,t}$  of absolute value of 1, and the corresponding persistent parameter in  $A$  equals  $\beta$ . This leads to a parameter identification problem. Below, I demonstrate this problem using a simple example based on investors only considering dividend yield as a predictor.

In this case, investors observe both dividend yields and past returns to extract shocks to expected returns  $\epsilon_\mu$  in order to update their return expectations. Assuming investors understand the present value relationship, their perceived system becomes

$$r_{t+1} = \mu_t + \epsilon_{d,t+1} - \rho\kappa_\mu\epsilon_{\mu,t+1} + \rho\kappa_g\epsilon_{g,t+1} \quad (9)$$

$$dp_{t+1} = (1 - \phi)B_{dp} + \phi dp_t + \kappa_\mu(\beta - \phi)\mu_t + \kappa_\mu\epsilon_{\mu,t+1} - \kappa_g\epsilon_{g,t+1} \quad (10)$$

$$\begin{pmatrix} \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix} \sim N \left[ 0, \begin{pmatrix} \sigma_d^2 & \sigma_{\mu d} & \sigma_{gd} \\ \sigma_{\mu d} & \sigma_\mu^2 & \sigma_{\mu g} \\ \sigma_{gd} & \sigma_{\mu g} & \sigma_g^2 \end{pmatrix} \right] \quad (11)$$

where  $\kappa_\mu = \frac{1}{1-\rho\beta}$  and  $\kappa_g = \frac{1}{1-\rho\phi}$  and  $\rho=0.94$  and  $B_{dp}$  is a constant.

The system in Equation (9) and (10) presents a parameter identification problem: investors must separate three shocks,  $\epsilon_{d,t}$ ,  $\epsilon_{\mu,t}$ , and  $\epsilon_{g,t}$ , from two observables, namely, the innovations in realized returns ( $u_{t+1}$ ) and the dividend-price ratios ( $v_{t+1}$ ), or

$$u_{t+1} = \epsilon_{d,t+1} - \rho\kappa_\mu\epsilon_{\mu,t+1} + \rho\kappa_g\epsilon_{g,t+1} \quad (12)$$

$$v_{t+1} = \kappa_\mu\epsilon_{\mu,t+1} - \kappa_g\epsilon_{g,t+1} \quad (13)$$

Because of this parameter identification issue, different investors can persistently disagree on their subjective value of expected return shocks and how their expected return process evolves. As these are not uniquely pinned down by the data, prior beliefs about parameter values in the

<sup>15</sup>Typically,  $R^2$  in regressions of future returns on predictors is small, and when the future returns are of short horizon, such as one year, the  $R^2$  is smaller than 10%. Furthermore, as discussed in [Welch and Goyal \(2008\)](#), the out-of-sample predictive powers of these predictors are also poor.

system are necessary to form a unique return expectation. In the next subsection, I discuss *which* prior beliefs need to be set subjectively by investors.

The parameter identification problem outlined here has been discussed in the literature; researchers who try to forecast future returns using Kalman filters face the same issue.<sup>16</sup> To avoid or solve this problem, the literature either simply imposes a value on the unidentified parameters or conducts a Bayesian analysis to examine what prior-belief values are the most accurate in terms of fitting the historical data.<sup>17</sup> Essentially, these exercises impose priors that constrain the reported return expectations.

Adding more predictors to the system will not completely resolve the problem, as other predictors are also imperfect and introduce more noise. For example, adding the consumption-wealth ratio (CAY) theoretically introduces shocks to the payout ratio and the leverage process. Even though aggregate consumption is correlated with aggregate dividends, the portion of consumption that is paid to shareholders as dividends varies over time. This new shock means the parameter identification problem persists.

### 2.3.2 What Drives Differences in Return Expectations?

I show that the two prior beliefs investors hold drive their return expectations: (i) how important expected cash flow news is for driving asset valuation when compared to discount rate news, and (ii) whether positive cash flow news affects future returns negatively or positively. Quantitatively, when investors believe asset prices are mostly driven by fundamentals, their return expectations are more likely to appear as extrapolative, rather than contrarian, and vice versa. I explain these results below and more analysis can be found in Appendix F.

To understand intuitively why these two particular priors can lead to procyclical and counter-cyclical return expectations, I consider extracting the latent expected return process from the dividend-price ratio alone, through

$$dp_t = B_{dp} + \frac{1}{1 - \rho\beta}\mu_t - \frac{1}{1 - \rho\phi}g_t \quad (14)$$

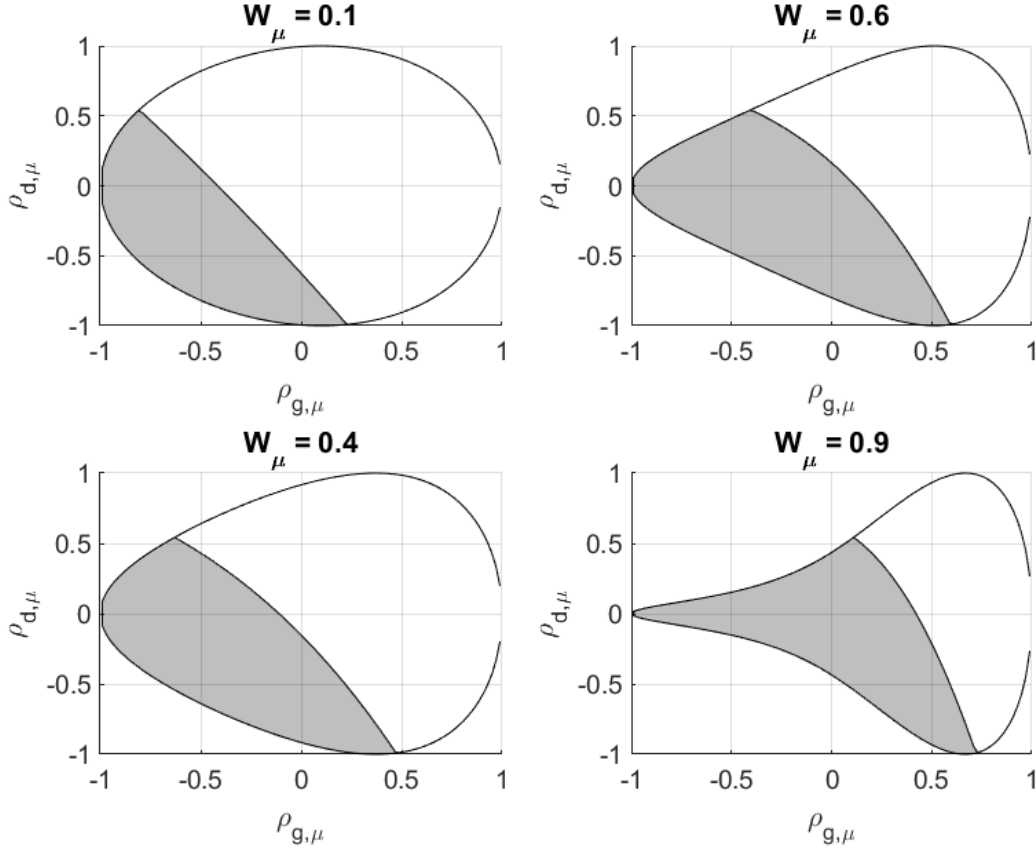
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<sup>16</sup>See for example: [Cochrane \(2008\)](#); [R. S. Kojen and Van Nieuwerburgh \(2011\)](#); [Pástor and Stambaugh \(2009\)](#); [Rytchkov \(2012\)](#).

<sup>17</sup>As an example, [Van Binsbergen and Kojen \(2010\)](#) assume the parameter  $\sigma_{gd}$  to be zero and [Pástor and Stambaugh \(2009\)](#) find that when econometricians require the correlations between  $u_{t+1}$  and  $v_{t+1}$  to be strongly negative, dividend yields have better performance in terms of forecasting future returns.

From the point of view of an econometrician, in order to distinguish the process  $\mu_t$  from the process  $g_t$ , they need to specify (i) how persistent the expected return process is compared to the cash flow process, and (ii) how the shocks to these two processes are correlated. In the case that an econometrician believes that most price-dividend moves are due to shocks to expected cash flow growth (because the cash flow process is much more persistent ( $\phi > \beta$ )), they would more likely believe that a positive change in the dividend-price ratio is a result of lowered expected future cash flow growth. Additionally, if they also believe that negative cash flows shocks are typically associated with negative future returns, they would lower their return expectations. In this case, their return expectations are negatively related to positive changes in the dividend-price ratio, therefore appearing to be procyclical.

Figure 3: *Who Will Be Contrarian?* Different Priors and Resulting Return Expectations



Note: The grey shaded areas represent the parameter space for  $(\rho_{d,\mu}, \rho_{g,\mu})$  in which an investor appears contrarian, or  $\tilde{m} < 0$ ; the white areas within the closed space in each graph are parameter spaces in which an investor appears extrapolative. Each subplot has a fixed level of (relative) discount rate volatility, defined as  $W_\mu = \frac{\kappa_\mu \sigma_\mu}{\sigma_{v,t}}$ . The area within the closed loops are feasible parameter spaces for  $(\rho_{d,\mu}, \rho_{g,\mu}, W_\mu)$  that satisfy the condition that the correlation matrix of  $(\rho_{d,\mu}, \rho_{v,d}, \rho_{v,\mu})$  needs to be semi-positive definite. This constraint puts bounds on the value of  $\rho_{d,\mu}$ , through  $\rho_{d,\mu} \in [-\sqrt{1 - \rho_{v,\mu}}, \sqrt{1 - \rho_{v,\mu}}]$ .

Figure 3 demonstrates how different priors would impact investors' subjective return expectations. More specifically, the figure plots the possible value pairs of priors on parameters  $(\rho_{d,\mu}, \rho_{g,\mu}, W_\mu)$  in order for a forecaster to appear contrarian or extrapolative.<sup>18</sup>  $W_\mu := \frac{\kappa_\mu \sigma_\mu}{\sigma_v}$  denotes the volatility of discount rate shocks (numerator) as a proportion to the shocks to dividend

<sup>18</sup>Notice  $W_\mu$  is not the same as the discount rate variation as a percentage of the total dividend yield variance. However, they are positively related to each other, up to scaling by the persistent parameters. Appendix F provides more detailed discussion on this subject.

yields (denominator). The area within the closed lines indicates the feasible parameter space for the parameters  $(\rho_{d,\mu}, \rho_{g,\mu}, W_\mu)$ , and the shaded area within each of the feasible regions is the parameter space in which an investor will appear to be contrarian.<sup>19</sup> The white area in each closed loop is the parameter space in which an investor will appear extrapolative. Each subplot in Figure 3 has a fixed level of (relative) discount rate volatility ( $W_\mu$ ).

The figure provides the following insights. First, investors will mostly likely appear extrapolative when they interpret (expected) cash flow news as positively related to future returns. Within each subplot, the upper right corner where  $\rho_{g,\mu}$  and  $\rho_{d,\mu}$  take on higher values, are regions in which investors expectations would appear extrapolative. This is intuitive, as investors are essentially extrapolating from current cash flow news to predict future returns, if cash flow news is important to them.

Second, the more an investor considers expected cash flow to be important for asset prices, the less likely the investor will appear to be contrarian. Quantitatively, when expected future cash flow is the dominant force (for example, as in the top left panel where  $W_\mu = 0.1$ , as long as  $\rho_{g,\mu} > 0.15$ ), investors would appear to be extrapolative, no matter how negative of a value for  $\rho_{d,\mu}$  they believe in. On the other hand, when the forecaster believes the discount rate is more important (bottom right plot), all investors who believe that  $\rho_{g,\mu} < 0.1$  will appear contrarian (gray area).

Finally, the figure also shows how the expectations framework can accommodate rich dynamics in return expectations. Even in the case where all investors believe asset prices are driven by fundamentals (top left panel of Figure 3), some could appear contrarian while the others appear extrapolative because of their different beliefs in  $\rho_{g,\mu}$ , for example. For a more technical discussion about how these parameters are related to return expectations, see Appendix F. In the next section, I use survey data to back out prior beliefs of different investors, as these parameters are important for differentiating asset pricing models.

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<sup>19</sup>Following the condition that

$$\rho_{d,\mu}^2 - 2\rho_{v,\mu}\rho_{v,d}\rho_{d,\mu} + (1 - \rho_{v,d}^2)(1 - \rho_{v,\mu}^2) \leq 0$$

and

$$\rho_{v,d} \approx 0$$

### 3 Identifying Prior Subjective Beliefs from Survey Data

Using subjective return expectation data, the framework allows for the identification of investors' prior beliefs, which are crucial assumptions in asset pricing models. First, I describe how to estimate the prior beliefs in this framework. Next, I apply the estimation methodology to selected survey data and discuss the implications of the estimates on asset pricing theories. Finally, I discuss what makes the prior beliefs different by providing evidence that personal experiences of sell-side analysts impact their return expectations.

#### 3.1 Estimation Framework

Thanks to the observable surveys, or Equation (17) below, all parameters governing investors' return expectation processes are identifiable, including those in the variance-covariance matrix in (2). We have the following systems of equations:

$$\hat{r}_{t+1} = \hat{\mu}_t + \epsilon_{\Delta d,t+1} - \rho\kappa_\mu(\beta)\epsilon_{\mu,t+1} + \rho\kappa_g(\phi_g)\epsilon_{g,t+1} \quad (15)$$

$$\hat{d}p_{t+1} = \phi_g \hat{d}p_t + \kappa_\mu(\beta - \phi_g)\tilde{\mu}_t + \kappa_\mu\epsilon_{\mu,t+1} - \kappa_g\epsilon_{g,t+1} \quad (16)$$

$$\hat{\mu}_{t+1}^A = \beta\hat{\mu}_t^A + L(\beta)\epsilon_{\mu,t+1} \quad (17)$$

$$\hat{x}_{t+1} = A\hat{x}_t + \epsilon_{x,t+1} \quad (18)$$

where the  $\hat{\cdot}$  denotes that variables are demeaned and the shocks follow the multivariate normal as in (19):

$$\begin{pmatrix} \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \\ \epsilon_{x,t+1} \end{pmatrix} \sim N \left[ 0, \begin{pmatrix} \sigma_d^2 & \sigma_{\mu d} & \sigma_{gd} & \sigma'_{dx} \\ \sigma_{\mu d} & \sigma_\mu^2 & \sigma_{\mu g} & \sigma'_{\mu x} \\ \sigma_{gd} & \sigma_{\mu g} & \sigma_g^2 & \sigma'_{gx} \\ \sigma_{dx} & \sigma_{\mu x} & \sigma_{gx} & \sigma_x^2 \end{pmatrix} \right] \quad (19)$$

I can now estimate the system based on maximum likelihood, where  $\kappa_\mu(\beta) = \frac{1}{1-\rho\beta}$ ,  $\kappa_g(\phi_g) = \frac{1}{1-\rho\phi_g}$ , and  $L(\beta) = \sum_{k=0}^3 \beta^k$ .  $\hat{\mu}_{t+1}^A$  are the observed (demeaned) 12-month return expectations (and the superscript "A" denotes a variable as annual), which are the quarterly return expectations  $\hat{\mu}_t$  rolling forward, following the dynamics of  $\hat{\mu}_t$ . The return expectations follow Equation (17),

because the demeaned quarterly return expectations follow

$$\hat{\mu}_{t+1} = \beta \hat{\mu}_t + \epsilon_{\mu,t+1}$$

Since the main interests are in the covariance matrix and the persistent parameters, I use demeaned returns and expectation data,  $\hat{r}_{t+1}$  and  $\hat{\mu}_{t+1}^A$ , respectively. More details about the estimation procedure is documented in Section G.

## 3.2 Estimation Results

### 3.2.1 Parameter Estimates

Table 5 presents the parameter estimates based on return expectations of sell-side analysts, pension funds (“Shiller Institutional”), and CFOs.<sup>20</sup> I chose to apply the estimation framework to these three series for the following reasons. First, they show relatively distinct correlations among each other as shown in Table 2a. Second, they are based on surveys with similar questions, so there are fewer scaling issues.<sup>21</sup>

As analyzed in the previous section, the key parameters are, first, the persistent parameters ( $\phi$  and  $\beta$ ), which impact the relative importance of discount rate news compared to cash flow news ( $W_\mu$ ); and second, the correlations between cash flows and expected returns ( $\rho_{g,\mu}$ ,  $\rho_{d,\mu}$ ). Differences in these values are crucial in determining how return expectations will appear.

The parameter estimates yield the following insights. First, all three types of investors believe the cash flow process to be persistent, and more persistent than the discount rate process. In particular, sell-side analysts believe the cash flow process has a persistence parameter of 0.93, the highest among the three investor types, although the differences among them are small. The persistence of the return expectations, however, differs significantly across participants.<sup>22</sup> Sell-side analysts return expectations (0.478) are much less persistent than those of the CFOs (0.604). Furthermore, the parameter estimates of  $\sigma_\mu$  show that sell-side analysts’ expectations

<sup>20</sup>See Appendix B for more details about how these data are constructed.

<sup>21</sup>The buy-side analyst surveys are based on a seven-year forecasting horizon while the data on households is based on a one-year horizon with the question of “What do you think is the percent chance that this one thousand dollar investment will increase in value in the year ahead, so that it is worth more than one thousand dollars one year from now?” Data on pension managers, sell-side analysts and CFOs are all based on the question of the percent increase in the prices of stocks.

<sup>22</sup>Notice that this parameter estimate is close to auto-correlation parameter estimates from a simple ordinary least squares (OLS) regression.

Table 5: Parameter Estimates Return Expectation Dynamics

Parameter estimates based on return expectations of sell-side analysts, pension funds, and CFOs. *Sell-Side Analyst* represents value-weighted sell-side analyst return expectations for the S&P 500 index. *Shiller Institutional* are the 12-month return expectations for the DJI from the Shiller Survey of investment managers. *CFO* are CFOs' return expectations from the Duke University survey.  $\phi$  and  $\beta$  are persistence parameters for dividend growth and the discount rate process, respectively.  $\sigma_\mu$ ,  $\sigma_d$  and  $\sigma_g$  are volatility parameters for discount rate shocks, unexpected dividend shocks and expected cash flow growth shocks, respectively.  $\rho_{d,\mu}$  is the perceived correlation between dividend shocks and future returns,  $\rho_{g,\mu}$  is the perceived correlation between cash flow growth shocks and future returns, and  $\rho_{d,g}$  is the perceived correlation between dividend shocks and cash flow growth shocks.  $W_\mu$  is implied perceived the discount rate shock volatility as a proportion of dividend yield shocks.  $\tilde{m}$  and  $\tilde{n}$  are implied parameters governing the subjective return expectations through Equation (8).  $\kappa_\mu = \frac{1}{1-\rho\beta}$ , and  $\kappa_g = \frac{1}{1-\rho\phi}$ .  $\rho_{v,\mu}$  is the correlation between dividend-price ratio and future returns.  $\sqrt{Q}$  is the volatility of subjective return expectation in steady-state. Appendix E provides exact definitions of the implied parameters.

| Investor                                           | Sell-Side Analyst   | Shiller Institutional | CFO              |
|----------------------------------------------------|---------------------|-----------------------|------------------|
| <b>Panel A: Estimates of Structural Parameters</b> |                     |                       |                  |
| $\phi$                                             | 0.929<br>(0.03)     | 0.920<br>(0.018)      | 0.918<br>(0.024) |
| $\beta$                                            | 0.478<br>(0.058)    | 0.674<br>(0.09)       | 0.604<br>(0.166) |
| $\sigma_d$                                         | 0.022<br>(0.002)    | 0.021<br>(0.002)      | 0.021<br>(0.002) |
| $\sigma_\mu$                                       | 0.014<br>(0.002)    | 0.010<br>(0.002)      | 0.005<br>(0.001) |
| $\sigma_g$                                         | 0.008<br>(0.002)    | 0.012<br>(0.002)      | 0.012<br>(0.002) |
| $\rho_{d,\mu}$                                     | -0.474<br>(0.201)   | -0.0152<br>(0.059)    | 0.156<br>(0.354) |
| $\rho_{d,g}$                                       | -0.00222<br>(0.308) | 0.121<br>(0.059)      | 0.127<br>(0.42)  |
| $\rho_{g,\mu}$                                     | -0.552<br>(0.134)   | 0.349<br>(0.07)       | 0.675<br>(0.165) |
| <b>Panel B: Implied Parameters</b>                 |                     |                       |                  |
| $W_\mu$                                            | 0.304               | 0.326                 | 0.152            |
| $\tilde{m}$                                        | -0.103              | 0.173                 | 0.059            |
| $\tilde{n}$                                        | 0.029               | 0.167                 | 0.019            |
| $\kappa_\mu$                                       | 1.816               | 2.731                 | 2.316            |
| $\kappa_g$                                         | 7.883               | 7.377                 | 7.303            |
| $\rho_{v,\mu}$                                     | 0.756               | -0.028                | -0.575           |
| $\sqrt{Q}$                                         | 0.010               | 0.012                 | 0.005            |

are much more volatile than those of CFOs', consistent with the results of [De la O and Myers \(2020\)](#). This leads to beliefs that discount rate news is less important in driving the variations in valuation ratios than cash flow news. Specifically, when  $W_\mu < 0.5$ , all three types of investors would be placed on the two left panels in Figure 3. Among different investors, CFOs' estimates of the importance of discount rates is the least among the three investor types, making them the most prone to extrapolation.

Second, the three investor types have very different estimates on how cash flow news impacts future returns, especially regarding what news about expected cash flow growth means for future returns ( $\rho_{g,\mu}$ ). Sell-side analysts, in particular, would revise their return expectations downward in light of positive news about cash flows ( $\rho_{g,\mu} = -0.552$  and  $\rho_{d,\mu} = 0 - 0.474$ ), while CFOs hold opposing beliefs ( $\rho_{g,\mu} = 0.675$ ,  $\rho_{d,\mu} = 0.156$ ). Pension funds managers' views are in between those of the CFOs and the sell-side analysts. Thus, the parameter estimates of  $\rho_{v,\mu}$  are consistent with the correlations in Table 2. Sell-side analysts' return expectations are countercyclical ( $\rho_{v,\mu} > 0$ ) while those of CFOs are procyclical ( $\rho_{v,\mu} < 0$ ) and pension fund managers' return expectations have little correlation with dividend yields. Relating the findings to Figure 3: CFOs' beliefs would place them in the upper right corner in the top left panel while sell-side analysts would be in the lower left gray area in the bottom left panel.

Further, the volatility parameters of the unexpected cash flow shocks,  $\sigma_d$ , are large and similar for different participants at about 2% per quarter. This is reasonable because the main difference between the return series and the price-dividend ratio series is due to unexpected cash flow shocks. As a result, investors should be able to almost identify the value of the unexpected cash flow shock volatility from the two series.

### 3.2.2 Subjective Variance Decomposition of Returns and Dividend-Price Ratios

These estimates also shed light on investors' beliefs about why returns and prices move, which is of great interest to researchers. Therefore, I compute the variance decomposition of price-dividend ratios and returns and present the results for the three types of investors in Tables 6 and 7.

Table 6: Variance Decomposition for (Log) Dividend to Price Ratios

This table shows the variance decomposition of the (log) price-dividend ratios for three investor groups implied by the expectation formation framework proposed here.  $\mu_t$  is the subjective expected returns.  $g_t$  is the subjective expected cash flow growth.  $Cov(\mu_t, g_t)$  is the covariance between the expected return and cash flow growth processes.  $Var(dp)$  is the variance of the (log) dividend price ratio. *Sell-side Analyst* represents the value-weighted sell-side analyst return expectations for the S&P 500 index. *Institutional* are the 12-month return expectations for the DJI from the Shiller Survey of investment managers. *CFO* are CFOs' year-over-year return expectations from the Duke University survey.

|                          | $\mu_t$ | $g_t$   | $-2Cov(\mu_t, g_t)$ | $Var(dp)$ |
|--------------------------|---------|---------|---------------------|-----------|
| <b>Sell-side Analyst</b> |         |         |                     |           |
| Variance                 | 0.37%   | 2.01%   | -0.34%              | 2.03%     |
| Portion of Returns       | 18.21%  | 98.73%  | -16.93%             | 100.00%   |
| <b>Institutional</b>     |         |         |                     |           |
| Variance                 | 0.73%   | 3.15%   | -1.84%              | 2.03%     |
| Portion of Returns       | 35.85%  | 154.80% | -90.65%             | 100.00%   |
| <b>CFO</b>               |         |         |                     |           |
| Variance                 | 0.10%   | 2.69%   | -0.75%              | 2.03%     |
| Portion of Returns       | 4.71%   | 132.23% | -36.94%             | 100.00%   |

Consistent with the magnitude of  $W_\mu$ , market participants believe variations in future cash flows are the dominant force in driving returns and asset prices, as opposed to discount rates, as shown in Table 6.<sup>23</sup> This is mainly due to the higher persistence of the cash flow expectation processes, as opposed to the volatility of the shocks, as is evident from the fact that shocks to cash flows take up a smaller portion of variance of returns (Table 7). I show in Appendix H that such decomposition is robust when using the analysts' own cash flow expectations directly, instead of the implied cash flow expectation in the estimation.

<sup>23</sup>In fact, this view is consistent with the argument put forward by [Bordalo, Gennaioli, Porta, and Shleifer \(2020\)](#), which indicates that a lot of stock market puzzles are driven by biased expectations about market fundamentals.

Table 7: Variance Decomposition for Quarterly Unexpected Returns

This table reports the variance decomposition of quarterly unexpected returns for three investor groups.  $Var(\epsilon_d)$  is the variance of current dividend shocks.  $Var(\epsilon_\mu)$  is the variance of expected discount rate shocks.  $Var(\epsilon_g)$  is the variance of expected future cash flow growth shocks.  $r_{t+1}$  is the quarterly return, and  $\mu_t$  is the expected return.  $Var(r_{t+1} - \mu_t)$  is the variance of unexpected returns.  $Cov(\mu_t, g_t)$  is the covariance between the expected return and cash flow growth processes and *Sell-side Analyst* represents the value-weighted sell-side analyst return expectations for the S&P 500 index. *Institutional* are the 12-month return expectations for the DJI from the Shiller Survey of investment managers. *CFO* are CFOs' year-over-year return expectations from the Duke University survey.

|                              | $Var(\epsilon_d)$ | $Var(\epsilon_\mu)$ | $Var(\epsilon_g)$ | $-2Cov(\epsilon_\mu, \epsilon_g)$ | $-2Cov(\epsilon_d, \epsilon_\mu)$ | $2Cov(\epsilon_d, \epsilon_g)$ | $Var(r_{t+1} - \mu_t)$ |
|------------------------------|-------------------|---------------------|-------------------|-----------------------------------|-----------------------------------|--------------------------------|------------------------|
| <b>Sell-side Analyst</b>     |                   |                     |                   |                                   |                                   |                                |                        |
| Value                        | 0.04%             | 0.05%               | 0.36%             | 0.16%                             | 0.05%                             | 0.01%                          | 0.60%                  |
| Portion of Returns           | 7.33%             | 9.00%               | 59.15%            | 26.03%                            | 7.96%                             | 1.82%                          | 100.00%                |
| <b>Shiller Institutional</b> |                   |                     |                   |                                   |                                   |                                |                        |
| Value                        | 0.04%             | 0.06%               | 0.62%             | -0.13%                            | 0.00%                             | 0.05%                          | 0.60%                  |
| Portion of Returns           | 6.58%             | 10.10%              | 102.62%           | -21.91%                           | 0.17%                             | 8.16%                          | 100.00%                |
| <b>CFO</b>                   |                   |                     |                   |                                   |                                   |                                |                        |
| Value                        | 0.04%             | 0.01%               | 0.64%             | -0.12%                            | -0.01%                            | 0.05%                          | 0.60%                  |
| Portion of Returns           | 6.60%             | 2.10%               | 106.30%           | -19.89%                           | -1.26%                            | 8.52%                          | 100.00%                |

In fact, as shown in Table 6, all of the market participants think that the level of expected returns and the cash flow expectations are positively correlated, that is, people believe a higher expected future fundamental growth is accompanied by a higher expected return. This positive correlation also holds for the short-term shocks to expectations, or  $Cov(\epsilon_\mu, \epsilon_g)$ , with the exception of sell-side analysts, who believe that these two shocks are negatively correlated at the quarterly frequency.

### 3.2.3 Which Predictors Are Important for Return Expectations?

I find that Shiller's cyclically adjusted price-to-earnings (CAPE) ratio seems to be the most influential predictor from the perspective of investors, although investors interpret it differently.

Table 8 shows how different market participants interpret the signals from different well-known predictors. Following the logic of the present value relation, investors could interpret a positive shock to price-fundamental ratios as a sign of either a higher future expected cash flow or lower future returns or both.

Panel A shows the correlation between shocks to different predictors and shocks to expected returns. Shiller's CAPE ratio is an important predictor considered by both sell-side analysts and

Table 8: Which predictors are important for investors? Correlations Between Innovations in Predictors and investors subjective expectations

This table reports the correlations between innovations in the predictors and both expected returns (panel A) and expected cash flow growth (panel B). CAY is the consumption-wealth ratio from Professor Martin Lettau's website. CAPE is the Schiller's cyclically adjusted price-earnings ratio.  $E_g$  is analysts' earnings growth forecast.  $\mu$  are expected returns, and  $g$  are expected cash flow growth.

| Investor                                                                                    | Sell-Side Analyst | Shiller Institutional | CFO               |
|---------------------------------------------------------------------------------------------|-------------------|-----------------------|-------------------|
| <b>Panel A: Correlation between Innovations in Predictors and Expected Returns</b>          |                   |                       |                   |
| $\rho_{CAY,\mu}$                                                                            | 0.395<br>(0.132)  | 0.018<br>(0.089)      | -0.334<br>(0.112) |
| $\rho_{CAPE,\mu}$                                                                           | -0.866<br>(0.031) | -0.076<br>(0.109)     | 0.614<br>(0.091)  |
| $\rho_{E_g,\mu}$                                                                            | -0.045<br>(0.161) | -0.055<br>(0.082)     | 0.114<br>(0.122)  |
| <b>Panel B: Correlation between Innovations in Predictors and Expected Cash Flow Growth</b> |                   |                       |                   |
| $\rho_{CAY,g}$                                                                              | -0.210<br>(0.147) | -0.257<br>(0.138)     | -0.295<br>(0.148) |
| $\rho_{CAPE,g}$                                                                             | 0.834<br>(0.046)  | 0.820<br>(0.048)      | 0.900<br>(0.024)  |
| $\rho_{E_g,g}$                                                                              | 0.445<br>(0.145)  | 0.296<br>(0.142)      | 0.299<br>(0.142)  |

CFOs, albeit with different signs. Next, for sell-side analysts, the consumption-wealth ratio, or CAY, is a positive predictor for future returns, while for CFOs and consumers, this measure is a negative predictor. Relatively speaking, analysts' earnings growth forecasts are less important in terms of forming return expectations.<sup>24</sup>

Panel B shows the correlation between shocks to different predictors and expected future cash flows. Contrasting with results on expected returns, people seem to interpret predictors similarly. In particular, a higher CAPE ratio and a lower CAY are interpreted as signs of higher future cash flows, while higher expected future earnings growth from analysts are signs of higher future cash flow growth, as expected.

These results are a confirmation of the model assumption: that different people might interpret the same predictor as being a different signal for future returns. Due to the parameter identification problem, this persistent difference in attitude towards predictors ultimately results in the differences in return expectations dynamics.

<sup>24</sup>Notice that although the sign is negative, the correlation between earnings growth expectation shocks and discount rate shocks is not significantly different from zero for sell-side analysts, which might seem to contradict the result from the model estimation on  $\rho_{g,\mu}$ . I further discuss this point in Appendix H.

### 3.3 Discussions

#### 3.3.1 Implications for Equilibrium Asset Pricing Theories

The empirical estimates of investors' subjective beliefs provide new moments on investors' beliefs to distinguish asset pricing models. Below, I discuss how these estimates are related to asset pricing models.

First, all three types of investors (sell-side analysts, institutional investors, and CFOs) believe that expected future cash flow is more persistent and is the dominant force driving asset prices and returns. This evidence is at odds with models where rational agents believe expected cash flows are independent and identically distributed ([Campbell and Cochrane \(1999\)](#), [Barberis et al. \(2015\)](#)).<sup>25</sup> On the other hand, the results support models in which agents believe there is a persistent component in the cash flow process. This includes the long-run risk models ([Bansal and Yaron \(2004\)](#); [Pohl, Schmedders, and Wilms \(2021\)](#)) and models featuring agents learning about fundamentals ([Collin-Dufresne et al. \(2016\)](#); [Nagel and Xu \(2019\)](#)). Additionally, the variance decomposition results reveal that all three types of investors seem to overestimate how much cash flow variation contributes to the variation in asset prices when compared to objective measures of return expectations, which show that discount rate variation should contribute the most to asset price variation.<sup>26</sup>

Second, investors beliefs vary, depending on their type, regarding how cash flow news impacts future returns. This finding supports models featuring heterogeneous agents in a long-run risk environment ([Pohl et al. \(2021\)](#)), and models involving parameter learning ([Collin-Dufresne et al. \(2017\)](#)). Of course, these results do not rule out the possibility that a subset of investors with a particular prior is driving asset prices, which would support models such as [Adam et al. \(2017\)](#) and [Nagel and Xu \(2019\)](#).

Why would different investors form different priors in the first place? Models featuring parameter learning, such as [Collin-Dufresne et al. \(2017\)](#), emphasize the micro-founded channel of personal experiences. Differences in personal experiences could lead to different risk appetites

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<sup>25</sup>In [Barberis et al. \(2015\)](#), the rational agents hold contrarian return expectations to accommodate the extrapolative demand of irrational traders. In a way, the finding that expectations of sell-side analysts are contrarian and countercyclical provides a micro-foundation for exploring who are these rational contrarian investors that understand that cash flows are i.i.d.

<sup>26</sup>In [Cochrane \(2011\)](#) and [R. S. Kojen and Van Nieuwerburgh \(2011\)](#), discount rate variations contribute to more than 100% of the dividend-price ratio variation. See [Table 15b](#) for a direct comparison between subjective and objective variance decomposition in this sample.

(Malmendier and Nagel (2011)) or different subjective beliefs about future economic variables (Malmendier and Nagel (2016)). Based on the current expectation formation framework, these two channels would lead to different return expectations in terms of being more or less contrarian, for example. I test this hypothesis next, taking advantage of the breadth of data on analyst expectations.

### 3.3.2 What Drives Differences in Prior Beliefs? The Role of Personal Experience

Using surveys from individual analysts, I find that more experienced analysts tend to be more contrarian. These results highlight individual experiences as one potential channel that drives differences in analysts' prior beliefs, and further supports models featuring parameter learning.

I run two variations of the following regression

$$\mu_{i,f,t}^a = \alpha_t + \alpha_f + bR_{f,t-6} + cX_{i,f,t} + \beta R_{f,t-6} * X_{i,f,t} + \epsilon_{i,f,t} \quad (20)$$

where  $\mu_{i,f,t}^a$  are the analyst-level return expectations (variation 1) or their deviations from the consensus (variation 2);  $R_{f,t-6}$  is the (one-month lagged) past six-month return of firm  $f$ ;  $\alpha_t$  and  $\alpha_f$  are time and firm fixed effects, respectively;  $X_{i,f,t}$  are analyst individual-level variables such as personal experiences, at the time the analyst is issuing the expected return for firm  $f$ .

The parameter estimate of interest is  $\beta$ , which measures how much *more contrarian* an analyst is when the analyst's personal experience variables  $X_{i,f,t}$  increase by one unit. The contrarian magnitude is measured in terms of deviation from the consensus return expectation, compared with other analysts issuing their return expectation for the same firm during the same month.

When selecting personal experience variables, I consider the literature that has documented the fact that people learn from personal experiences (Malmendier and Nagel (2011), Malmendier and Nagel (2016)) and that information rigidity and sticky expectations also come into play (Mankiw and Reis (2002), Bouchaud, Krueger, Landier, and Thesmar (2019)). I use the number of months an analyst experiences recession, the number of years of experience an analyst has, as well as the number of stocks an analyst covers. To that end, I construct a comprehensive analyst-level data set on return and earnings expectations, which I document in more detail in Internet Appendix I.5.

Table 9: Analyst No. of years in the industry and return expectations

This table investigates the effect of analyst experience on their subjective return expectations. *Deviation from Consensus* is the analyst's own expected return subtracted from the firm-level mean consensus expected return measured from the week before the issuance of the analyst's own expected returns. *No Yrs Experience* measures, at the time an analyst issues an expected return for stock  $f$ , the number of years since the analyst has first issued a forecast (EPS/Price Target). *No. Firms Covered by Analyst* is the number of stocks the analyst covers. *No Months Recession* is the number of months of an analyst's career that have been during recessions. *Past 6m log.ret* are the one-month lagged six-month returns. \* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$ .

|                                     | <i>Dependent variable:</i> |                       |                       |                       |
|-------------------------------------|----------------------------|-----------------------|-----------------------|-----------------------|
|                                     | Deviation From Consensus   |                       | Expected Returns      |                       |
|                                     | (1)                        | (2)                   | (3)                   | (4)                   |
| No Yrs Experience                   | −0.042<br>(0.053)          | −0.050<br>(0.053)     | −0.009<br>(0.053)     | −0.006<br>(0.052)     |
| No Firms Covered by Analyst         | 0.022**<br>(0.009)         | 0.022**<br>(0.009)    | 0.028***<br>(0.009)   | 0.029***<br>(0.009)   |
| No Months Recession                 | 0.026<br>(0.017)           | 0.029*<br>(0.017)     | 0.021<br>(0.017)      | 0.022<br>(0.017)      |
| Past 6m log.ret                     | 0.132<br>(0.250)           | 0.469*<br>(0.256)     | −3.853***<br>(0.522)  | −3.321***<br>(0.524)  |
| Past 6m log.ret x No Yrs Experience | −0.072***<br>(0.022)       | −0.095***<br>(0.024)  | −0.135***<br>(0.036)  | −0.217***<br>(0.034)  |
| Including Recession Months          | Yes                        | No                    | Yes                   | No                    |
| Firm Fixed Effects?                 | No                         | No                    | Yes                   | Yes                   |
| Month Fixed Effects?                | Yes                        | Yes                   | Yes                   | Yes                   |
| Observations                        | 1,280,473                  | 1,054,913             | 1,332,314             | 1,096,782             |
| R <sup>2</sup>                      | 0.024                      | 0.017                 | 0.353                 | 0.392                 |
| Adjusted R <sup>2</sup>             | 0.024                      | 0.017                 | 0.348                 | 0.387                 |
| Residual Std. Error                 | 27.697 (df = 1280264)      | 24.545 (df = 1054743) | 29.265 (df = 1323792) | 25.821 (df = 1088634) |

Table 9 shows the estimates for Equation (20), in which the interacting variable with past returns is number of years of experience an analyst has had up to the time of issuance.<sup>27</sup> The left two columns consider the deviation from consensus as a dependent variable while the right two columns use expected returns. Within each group of the same dependent variables, the right column regression excludes the recession months as defined by the National Bureau of Economic Research (NBER).

Across different specifications, the estimates on  $\beta$  are significantly negative. Since the standard deviation of the analyst number of years of experience variable is about seven years, the  $\beta$  estimate in Column 1 can be interpreted as a more experienced analyst (with one or more years experience in the industry) would be lower than the consensus by 7.2%, compared to the analyst with less experience, when issuing estimates for the same firm during the same fiscal quarter, given the same level of past six months cumulative returns. These results are consistent with the hypothesis that analysts learn from the markets and adapt their expectations over time.

## 4 Conclusion

Investors' subjective return expectations play a central role in in asset pricing, yet our understanding of them is limited. Recent evidence of extrapolative return expectations based on surveys seems to paint some investors as irrational. My contribution here is to document contrarian expectations of Wall Street analysts and provide a bounded-rational interpretation for these heterogeneous return expectations. Furthermore, I demonstrate how the expectation formation framework developed here together with survey data provides new empirical moments to distinguish asset pricing models.

Future research can extend the current study to further our understanding about investors' decision process and asset prices. For example, more can be done to collect surveys, especially on the buy-side. [Dahlquist and Ibert \(2021\)](#) made progress on this front by collecting a large set of return expectations from asset management companies. Furthermore, connecting the heterogeneous beliefs to investors' heterogeneous institutional demand ([R. S. J. Koijen and Yogo \(2019\)](#)) is a logical next step. As an example, [Lochstoer and Tetlock \(2021\)](#) studies how closed-

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<sup>27</sup>In unreported tables, I also consider interacting the past realized returns with the number of firms covered, as well as the number of months in recession. These two variables do not show up as statistically significant when interacted with past returns.

end mutual funds' holdings and pricing premiums are related to investors' beliefs. Finally, what are the implications to asset prices if we take seriously the difference in implied subjective beliefs and objective values of structural parameters, such as the persistence in the cash flow process? As a start, in [Renxuan \(2020\)](#), I explore the idea that investors underestimate the dynamics of discount rates, which is supported by the results presented in Table 6, and find that this mispricing can explain many of the asset pricing anomalies in the cross-section.

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## **Appendix A More Details about Sell-Side Analysts' Return Expectations Data**

### **A.1 Measuring Analyst Return Expectations Using Analyst Price Targets**

Firm- and market-level analyst return expectations are constructed using a bottom-up approach based on analyst-level return expectations per analyst issuance.

I collect single issuance of price targets from individual analysts' 12-month<sup>28</sup> price targets for individual firms from the Institutional Brokers Estimate System (IBES) unadjusted data base and match it with the closing price from the Center for Research in Security Prices (CRSP) on the date the price target is issued<sup>29</sup> to compute return expectations with price targets for individual firms. The expected returns are computed by dividing an analyst's price target by the daily closing price on the day the estimates were issued and then subtracting the number 1<sup>30</sup>:

$$\mu_{i,f,d}^A = \frac{P_{i,f,d}^{A,12}}{P_{f,d}} - 1$$

where  $P_{i,f,d}^{A,12}$  is the price target of analyst  $i$  for firm  $f$ , issued at day  $d$ . The superscript 12 denotes the 12-month ahead estimates. Notice this methodology ensures there is no mechanical relation between the mean estimated expected returns and the level of prices. On each issuing date, the analyst has the freedom to pick their own price target since they observe the prices.

Firm-level return expectations are constructed together with the stop file provided by IBES to ensure individual estimates are not stale. IBES keeps track of the activeness of the individual estimates and provides a stop file for price targets.<sup>31</sup> I merge the point-in-time, analyst-level expected return file with the stop file on price targets to exclude estimates that analysts and IBES have confirmed to be no longer valid. Furthermore, to avoid stale estimates, I additionally restrict the estimates to be no older than 90 days when entering mean consensus estimates.<sup>32</sup>

I construct weekly firm-level consensus expected returns by taking the mean of all active analyst-level forecasts, although using the median makes no discernible difference for the main results. I drop analyst-level estimates that are greater than five standard deviations away from the mean estimates, and I winsorize the entire analyst-level database by 1% and 99% before calculating the firm-level consensus. I take the mean of the available expected return estimates

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<sup>28</sup>Other horizons are available, though the coverage is poor.

<sup>29</sup>In case the issuance date is a weekend, the last Friday prices are used. When the issuance is a holiday, the previous business day closing prices are used.

<sup>30</sup>The same formula is used in [Brav and Lehavy \(2003\)](#) and [Da and Schaumburg \(2011\)](#)

<sup>31</sup>According to IBES, this stop file "includes stops applied to estimates that are no longer active. This can result from several events, e.g. an estimator places a stock on a restricted list due to an underwriting relationship or the estimator no longer covers the company. Prior to June 1993, actual stop dates did not exist in the archive files used to create the Detail History. An algorithm was developed to determine the date when an estimate became invalid if, for example, a merger between companies occurred or an analyst stopped working for a firm, etc. Estimate that are not updated or confirmed for a total of 210 days, the estimate is stopped."

<sup>32</sup>[Engelberg, McLean, and Pontiff \(2019\)](#) allows the estimates to be at most 12 months old, in case the estimates are not covered by the stop file, although the choice makes little difference for the main results

for each firm by the end of Saturday each week, or

$$\mu_{f,w}^A = \sum_i \mu_{i,f,w}^A / I_f$$

where  $I_f$  is the number of analysts for firm  $f$  at week  $w$ . For most of the application of the paper, I use firm-level return estimates based on monthly data, which is the consensus data on the last Saturday before each calendar month end.

Market-level aggregate return expectations are constructed based on the Standard and Poor's S&P 500 universe. The aggregate market-level return expectations for the S&P 500 index is the firm market-cap ( $M_{f,t}$ ) weighted average of firm-level return expectations at the end of the month  $t$ , or

$$\mu_{m,t}^A = \sum_f \frac{M_{f,t-1}}{\sum_f M_{f,t-1}} \mu_{f,t}^A$$

In untabulated results, I also examined the results based on an equal-weighted index. The results do not change qualitatively.

Additionally, the firm-level, 12-month forward earnings-to-price ratio is constructed based on IBES analysts' first fiscal year and second fiscal year-ahead earnings per share (EPS) estimates. In order to compute total earnings, I use analyst-level, detailed, unadjusted EPS estimates multiplied by the number of shares outstanding at the date when the analyst issued the EPS estimates. Subsequently, firm-level earnings estimates for 12 months ahead are linearly interpolated between the one- and two-year-ahead median earnings estimates for the firm at each month end. This methodology is consistent with how CRSP constructs their indices and is also used in [De la O and Myers \(2020\)](#).

## Appendix B Details on the Sources of Return Expectation Data

As an overview, for analyst earnings and return forecast data, I use the IBES unadjusted file. Firm fundamentals and S&P 500 membership data are from COMPUSTAT and daily pricing data are from the CRSP. The CFO return expectation data is from Duke University CFO Global Business Outlook available on their website.<sup>33</sup> Retail investor return expectations are based on Robert

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<sup>33</sup><https://www.cfosurvey.org/>

Shiller's surveys<sup>34</sup> as well as the consumer survey conducted by the University of Michigan.<sup>35</sup>

For objective measures of expected returns, I construct aggregate price-dividend ratio using the total returns and price returns on S&P 500 index. I document the detail of the construction in Internet Appendix I.4. For the aggregate price-earning data, I used the data kindly provided by Professor Robert Shiller on his website. The consumption-wealth ratio, or CAY is downloaded from Professor Martin Lettau's website, which is constructed based on [Lettau and Ludvigson \(2004\)](#). Below are more details about these data sources.

**Shiller Survey** asks participants about their expected percentage increase on the Dow-Jones index over different horizons in the future. The participants consist of two groups: one retail investor, who is randomly selected U.S. wealthy individuals; and the other is an institution, or "the investment managers section of the Money Market Directory of Pension Funds and Their Investment Managers". I use the data set which aggregates the raw data and report the "the percent of the population expecting an increase in the Dow in the coming year" from July 2001 to Dec 2020, during which the data was collected monthly and the moving average of the six-month data are published. I also consider a data set that is used and made public by [Adam et al. \(2017\)](#), which uses the raw averages of expected price growth from the Shiller Survey, but only available from 2001-Q1 to 2012-Q4.

**The Michigan survey** asks about 500 households in the U.S. "What do you think is the percent chance that this one thousand dollar investment will increase in value in the year ahead, so that it is worth more than one thousand dollars one year from now?" and calculates the average across all responses. The survey is conducted monthly. I use data that starts in August, 2002 and ends in December, 2018. I mainly use quarterly data at the end of each calendar quarter to be consistent with the other survey data.

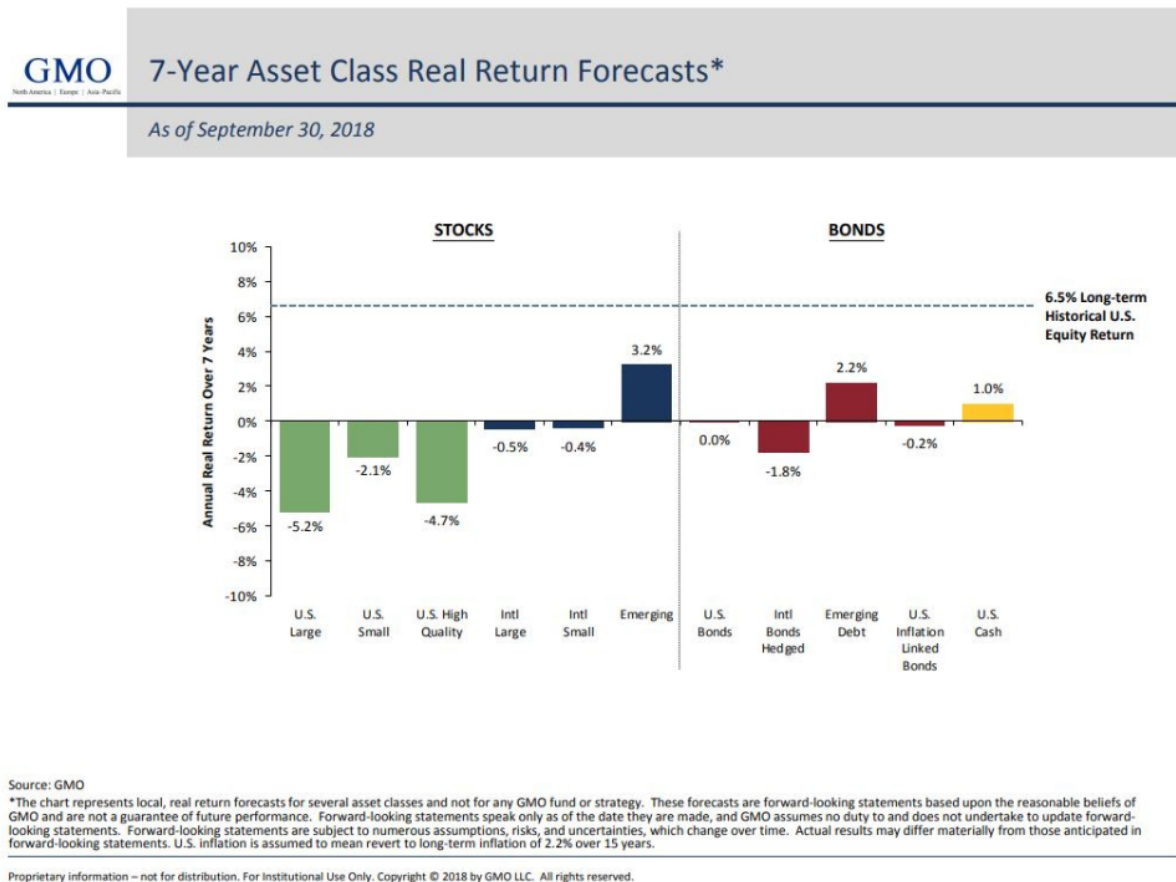
**GMO 7-year Asset Class Forecasts** is produced quarterly by GMO, which consists of return forecasts for 7-year ahead for different segments of equity and bond markets, including but not limited to U.S. large caps, international small caps, and emerging market bonds. The data after 2017 are available directly on GMO's website. For pre-2017 data, I hand collected the data from the internet. This is possible because the company publishes the return forecasts dating back

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<sup>34</sup>Available on Yale University's website: <https://som.yale.edu/faculty-research-centers/centers-initiatives/international-center-for-finance/data/stock-market-confidence-indices/united-states-stock-market-confidence-indices>

<sup>35</sup>Available on <https://data.sca.isr.umich.edu/data-archive/mine.php>

Figure 4: A Snapshot of One GMO 7-year Asset Class Forecast



from the second quarter of 2000 and the publication is in the form of a standalone image, as shown in Figure 4. On websites of advisers or consultants, they have cached historical figures of these snapshots. However, I was not able to collect the full history of their return forecasts in quarterly frequency.

Table 10: Summary Statistics Subjective Quarterly Expectations

This table displays summary statistics for the quarterly subject return expectations. *ER.analyst* are value-weighted sell-side analyst return expectations for the S&P 500 index. *LTG.analyst* is the analyst long-term growth expectation for the S&P 500 index. *ER.CFO* are CFOs' year-over-year return expectations from the Duke University survey. *ER.consumer(raw)* are the percentage of retail investors in the Shiller Survey who expect the DJI to increase in the next year. *ER.consumer(proj)* represents one-year return expectations from the Michigan Survey of households, projected onto the objective expected returns based on dividend-price ratio and CAY. *ER.buy.side* are from GMO's equity forecasts. *ER.Shiller.12m* is the aggregate expected return from the two Shiller Surveys. *ER.GMO.7y* are from GMO's equity forecasts. *Shiller.Inst.Pct.Up* and *Shiller.Ind.Pct.Up* are the percentage of institutional and individual investors, respectively, in the Shiller Survey expecting an increase in the DJI over the next year. *Michigan.Pct.Up* is the percentage of households in the Michigan survey that expect an increase in the DJI over the next year. *ER.SPF.Pct.10yr* are from the Federal Reserve Survey of Professional Forecasters 10-year stock return data. *No.IPO* is the number of initial public offerings during the quarter. *Equity.Net.Issuance* is the gross issuance subtracted by the equity retirement. *totl.ret.qtrly.SP500* is the quarterly total return of the S&P 500 index. More information on the data sources can be found in Appendix B.

| Statistic            | N  | Mean    | St. Dev. | Min    | Pctl(25) | Pctl(75) | Max    |
|----------------------|----|---------|----------|--------|----------|----------|--------|
| ER.analyst           | 70 | 0.143   | 0.037    | 0.095  | 0.114    | 0.161    | 0.270  |
| LTG.analyst          | 70 | 0.115   | 0.015    | 0.084  | 0.105    | 0.122    | 0.153  |
| ER.CFO               | 69 | 0.056   | 0.014    | 0.022  | 0.046    | 0.065    | 0.091  |
| ER.consumer(raw)     | 66 | 53.117  | 6.290    | 36.500 | 48.000   | 57.400   | 63.600 |
| ER.consumer(proj)    | 66 | 0.041   | 0.023    | 0.002  | 0.025    | 0.060    | 0.102  |
| ER.buy.side          | 70 | -0.0002 | 0.025    | -0     | -0.02    | 0.01     | 0      |
| ER.Shiller.12m       | 46 | 0.051   | 0.021    | 0.018  | 0.036    | 0.068    | 0.108  |
| ER.GMO.7y            | 70 | -0.0002 | 0.025    | -0     | -0.02    | 0.01     | 0      |
| Shiller.Inst.Pct.Up  | 70 | 77.917  | 5.689    | 62.810 | 74.797   | 81.385   | 92.520 |
| Shiller.Ind.Pct.Up   | 70 | 77.293  | 8.486    | 61.270 | 71.978   | 84.220   | 95.280 |
| Michigan.Pct.Up      | 66 | 53.117  | 6.290    | 36.500 | 48.000   | 57.400   | 63.600 |
| ER.SPF.Pct.10y       | 70 | 6.735   | 0.796    | 5.337  | 6.152    | 7.437    | 7.683  |
| No.IPO               | 70 | 42.914  | 22.122   | 1      | 31       | 61.8     | 85     |
| Equity.Net.Issuance  | 70 | -0.048  | 0.033    | -0.150 | -0.071   | -0.031   | 0.033  |
| totl.ret.qtrly.SP500 | 70 | 0.020   | 0.078    | -0.219 | -0.013   | 0.063    | 0.159  |

Table 11: Correlations Between Different Subjective Expectations

This table reports correlations between the different subjective return expectations. *ER.analyst* are value-weighted sell-side analyst return expectations for the S&P 500 index. *LTG.analyst* is the analyst long-term growth expectation for the S&P 500 index. *ER.CFO* are CFOs' year-over-year return expectations from the Duke University survey. *ER.Shiller.12m* is the aggregate expected return from the two Shiller Surveys. *ER.GMO.7y* are from GMO's equity forecasts. *Shiller.Inst.Pct.Up* and *Shiller.Ind.Pct.Up* are the percentage of institutional and individual investors, respectively, in the Shiller Survey expecting an increase in the DJI over the next year. *Michigan.Pct.Up* is the percentage of households in the Michigan survey that expect an increase in the DJI over the next year. *ER.SPF.Pct.10yr* are from the Federal Reserve Survey of Professional Forecasters 10-year stock return data. *No.IPO* is the number of initial public offerings during the quarter. *Equity.Net.Issuance* is the gross issuance subtracted by the equity retirement. *tot.ret.qtrly.SP500* is the quarterly total return of the S&P 500 index. More information on the data sources can be found in Appendix B.

|                     | ER.analyst | LTG.analyst | ER.CFO  | ER.Shiller.12m | ER.GMO.7y | Shiller.Inst.Pct.Up | Shiller.Ind.Pct.Up | Michigan.Pct.Up | ER.SPF.Pct.10y | No.IPO  | Equity.Net.Issuance | tot.ret.qtrly.SP500 |
|---------------------|------------|-------------|---------|----------------|-----------|---------------------|--------------------|-----------------|----------------|---------|---------------------|---------------------|
| ER.analyst          | 1.00       |             |         |                |           |                     |                    |                 |                |         |                     |                     |
| LTG.analyst         | 0.26**     | 1.00        |         |                |           |                     |                    |                 |                |         |                     |                     |
| ER.CFO              | -0.27**    | 0.29**      | 1.00    |                |           |                     |                    |                 |                |         |                     |                     |
| ER.Shiller.12m      | 0.00       | 0.19        | 0.68*** | 1.00           |           |                     |                    |                 |                |         |                     |                     |
| ER.GMO.7y           | 0.41***    | -0.39***    | -0.06   | -0.20          | 1.00      |                     |                    |                 |                |         |                     |                     |
| Shiller.Inst.Pct.Up | 0.21*      | 0.03        | 0.20*   | 0.33**         | 0.26**    | 1.00                |                    |                 |                |         |                     |                     |
| Shiller.Ind.Pct.Up  | 0.37***    | 0.20*       | 0.47*** | 0.65***        | 0.34***   | 0.57***             | 1.00               |                 |                |         |                     |                     |
| Michigan.Pct.Up     | -0.68***   | 0.38***     | 0.38*** | 0.04           | -0.69***  | -0.37***            | -0.27**            | 1.00            |                |         |                     |                     |
| ER.SPF.Pct.10y      | 0.45***    | 0.21*       | 0.35*** | 0.30**         | 0.53***   | 0.50***             | 0.81***            | -0.31**         | 1.00           |         |                     |                     |
| No.IPO              | -0.59***   | 0.11        | 0.20*   | -0.22          | -0.31***  | -0.22*              | -0.24*             | 0.66***         | -0.12          | 1.00    |                     |                     |
| Equity.Net.Issuance | -0.01      | 0.05        | 0.45*** | 0.49***        | 0.01      | 0.16                | 0.29**             | -0.12           | 0.04           | -0.27** | 1.00                |                     |
| tot.ret.qtrly.SP500 | -0.60***   | -0.18       | 0.44*** | 0.16           | 0.06      | -0.05               | 0.03               | 0.15            | -0.05          | 0.29**  | 0.32***             | 1.00                |

## Appendix C Detailed Analysis on Sell-Side Analysts' Contrarian Return Expectations

### C.1 Firm-level Results

The results in Table 4 show the analyst's contrarian views are present at the market level, which is a value-weighted average of firm-level variables. At the firm level, lower past returns are typically associated with an increase in valuation ratio, such as book-to-market ratio. An extensive literature documents a positive relation between valuation ratios and future stock returns. This raises the question of whether the negative relationship between past returns and analyst expectations is merely an effect of analysts using firms' valuation ratios as a determinant in forming their expectation.

To answer this question, I run firm-level analyst return expectations on past returns together with firm-level characteristics, including valuation ratios, as control variables

$$\mu_{i,t}^A = \alpha_i + bR_{i,t-2,t-6} + cX_{i,t-2} + e_{i,t} \quad (21)$$

Since the paper focuses on the *time-variation* of analyst return expectation, I include a firm effect in the panel regression.

First, the results show that analysts also hold strong contrarian view at the firm level. In fact, when only including past 6-month returns (Column 1), the coefficient on analyst contrarian view has an estimate of -0.11, very close to the results on the aggregate, shown in Table 4.

Second, the coefficient on past returns changes very little when including other control variables, as shown in Column 3 and 4. Firm valuation ratios such as book-to-market ratio do predict analysts' return expectations, as shown in Column 2, although the economic magnitude is much smaller, when compared to past returns.

Interestingly, Column 5 also show that analyst's own forecasts on future earnings, both one-year ahead and long-term, have a strong correlation with their own return expectation. This is consistent with the results documented in [Da, Hong, and Lee \(2016\)](#). Furthermore, the higher a firm's investment, the higher analysts would expect its future expected returns to be. To the best of my knowledge, there is not other prior literature documenting the effect of investment

Table 12: Cross-Sectional Determinants of Analyst Return Expectations

This table reports firm-level analyst return expectation regressed on past returns together with other of firm-level characteristics together with control variables in  $X_{i,t-2}$

$$\mu_{i,t}^A = \alpha_i + bR_{i,t-2,t-6} + cX_{i,t-2} + e_{i,t}$$

with a firm fixed effect  $\alpha_i$ . Sample is based on S&P 500 firms from 2002-01-01 to 2018-12-31. Standard errors are clustered by firm and month. *lag.2m* means variables are lagged by 2 months. *cum.6m.ret* is cumulative total returns for the firm in the past 6 months; *CF/P* is cash flow to market cap; *B/M* is book-to-market ratio; *fwd.12m.E/P* is analysts' 1-year ahead forward earnings divided by market cap; *LTG* is analyst long-term growth estimates; *Prof* is operating profitability defined as in Fama and French (2006); *Inv* is annual asset changes divided by assets, as defined in Fama and French (2006). Firm-level variables are winsorized at 1% and 99% over the entire sample. \*p < 0.1; \*\*p < 0.05; \*\*\*p < 0.01.

|                         | Dependent variable:         |                     |                      |                      |
|-------------------------|-----------------------------|---------------------|----------------------|----------------------|
|                         | Analyst Return Expectations |                     |                      |                      |
|                         | (1)                         | (2)                 | (3)                  | (4)                  |
| lag.2m.cum.6m.ret       | -0.111***<br>(0.011)        |                     | -0.102***<br>(0.010) | -0.094***<br>(0.010) |
| lag.2m.CF/P             |                             | -0.014<br>(0.012)   |                      |                      |
| lag.2m.B/M              |                             | 0.024***<br>(0.005) | 0.013***<br>(0.004)  | 0.014***<br>(0.004)  |
| lag.2m.fwd.12m.E/P      |                             |                     |                      | 0.519***<br>(0.074)  |
| lag.2m.LTG              |                             |                     |                      | 0.351***<br>(0.025)  |
| lag.2m.Prof             |                             |                     |                      | -0.021***<br>(0.006) |
| lag.2m.Inv              |                             |                     |                      | 0.097***<br>(0.014)  |
| Constant                | 0.139***<br>(0.003)         | 0.123***<br>(0.003) | 0.133***<br>(0.004)  | 0.059***<br>(0.007)  |
| Observations            | 99,716                      | 91,202              | 92,735               | 86,184               |
| R <sup>2</sup>          | 0.049                       | 0.012               | 0.052                | 0.138                |
| Adjusted R <sup>2</sup> | 0.049                       | 0.012               | 0.052                | 0.138                |
| Residual Std. Error     | 0.105 (df = 99714)          | 0.108 (df = 91199)  | 0.105 (df = 92732)   | 0.099 (df = 86177)   |

on subjective return expectations. However, this is not the focus of the current paper so I will not explore this further.

In sum, the pattern that past returns are strong predictors for analysts' return expectations are robust at the firm level and are not due to analysts using firms' valuation ratios to make forecasts on firms' future returns.

## C.2 Concerns with Stale Estimates? Return Expectations of Analysts' First-time Issuances

One concern regarding the conclusion that analysts hold contrarian views is that analysts' stale price targets might be driving the negative relation between past returns and future analyst return expectations. To illustrate the concern more clearly, consider an extreme case where

analysts never change their price targets. As prices go up, the expected returns go down mechanically and the contrarian conclusion follows. Although this might still be due to analysts' intentionally holding slow-moving return expectations and thus appearing to be contrarian, it may come from analysts' limited attention span or laziness.

To eliminate such concerns, I show that the contrarian results are robust to a sample containing only return expectations based on each analyst's first-time issuance ever for a particular firm in the entire IBES data base. Because the first-time issuance is always fresh and there is no potential staleness due to the aggregation process, such results mean the negative correlation between analyst return expectations and past returns are mainly driven by the analyst's contrarian views rather than the staleness of analyst forecasts.

Table 13 shows the results for the following regression

$$\mu_{j,i,t}^A = a + bR_{i,t-1,t-6} + cX_{i,t-1} + e_{j,t} \quad (22)$$

where  $j$  denotes an analyst and  $i$  denotes a firm. In particular,  $\mu_{j,i,t}^A$  is the first estimate a particular analyst ever issued for a particular firm for both the EPS and price target data bases.<sup>36</sup> Notice the analyst's issuances are recorded on the day of the issuance within each month and subsequently pushed to the end of the month to be run on monthly data. Therefore, to avoid look-ahead bias, I require the independent variables to enter the regression with a one-month lag, so the predictive regression is entirely out of sample. I calculate standard errors by clustering by firm and month.

Results in Table 13 show that the contrarian results documented at the aggregate and firm level also hold for the analyst-level regression. Coefficients on the past returns are statistically negative for both the entire IBES and the S&P 500 universe. Furthermore, the magnitude of the coefficients across all specifications is very similar to those in the aggregate-level and firm-level regression, ranging from -0.12 to -0.17. Other firm-level controls do not eliminate the contrarian effect.

In sum, these results confirm that staleness and other mechanical reasons are not the force driving the negative coefficients, and support the conclusion that analysts' hold contrarian return

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<sup>36</sup>EPS forecasts go back much further than price target data, which start to have good quality data from early 1980 and 2000, respectively. The reason for also considering the EPS data base is to avoid cases in which an analyst has potentially stale price target estimates which are not reported in the price target data base. For more details regarding how the two data bases are merged, see Internet Appendix I.5.

expectations.

Table 13: Analyst-level first-time-issued return expectations vs. past returns

An analyst's first-ever issued return expectation is regressed on the firm-level monthly variables including (lagged one-month) cumulative past six-month returns and other control variables. An analyst's first-ever return expectation is based on the analyst's first price targets ever issued in both EPS and price target data in the IBES data base. *B/M*, *Inv*, and *OpIB* are book-to-market, investment, and operating profitability variables, respectively, as defined in [Fama and French \(2006\)](#). Sample period: 2002-01-01 to 2018-12-31. Independent variables are winsorized at 1% and 99%. \* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$ . SEs are clustered by firm and month.

|                         | Dependent variable:                                     |                      |                      |                      |
|-------------------------|---------------------------------------------------------|----------------------|----------------------|----------------------|
|                         | Analyst First-Ever Issued Return Expectation For A Firm |                      |                      |                      |
|                         | (1)                                                     | (2)                  | (3)                  | (4)                  |
| past 6m.ret             | −0.166***<br>(0.022)                                    | −0.148***<br>(0.020) | −0.126***<br>(0.022) | −0.125***<br>(0.023) |
| B/M                     |                                                         | −0.013<br>(0.015)    |                      | −0.006<br>(0.015)    |
| Inv                     |                                                         | 0.207***<br>(0.037)  |                      | 0.222***<br>(0.042)  |
| OpIB                    |                                                         | −0.220***<br>(0.017) |                      | −0.033**<br>(0.014)  |
| Constant                | 0.251***<br>(0.009)                                     | 0.272***<br>(0.013)  | 0.155***<br>(0.007)  | 0.154***<br>(0.012)  |
| Universe                | All IBES                                                | All IBES             | SP500                | SP500                |
| Observations            | 9,282                                                   | 8,310                | 2,887                | 2,795                |
| R <sup>2</sup>          | 0.032                                                   | 0.107                | 0.029                | 0.052                |
| Adjusted R <sup>2</sup> | 0.032                                                   | 0.107                | 0.029                | 0.051                |
| Residual Std. Error     | 0.335 (df = 9280)                                       | 0.299 (df = 8305)    | 0.189 (df = 2885)    | 0.187 (df = 2790)    |

## Appendix D Simulating Return Expectations

To show that the model proposed has the ability to capture the key empirical moments for both return predictability as well as the heterogeneous return expectation dynamics, I conduct simulation exercises.

I first simulate 500 quarters of data based on the system from Equation (3) to (7). Panel A in Figure 5 shows the simulated data over time, which plots the predictor  $x_t$  as dividend-price

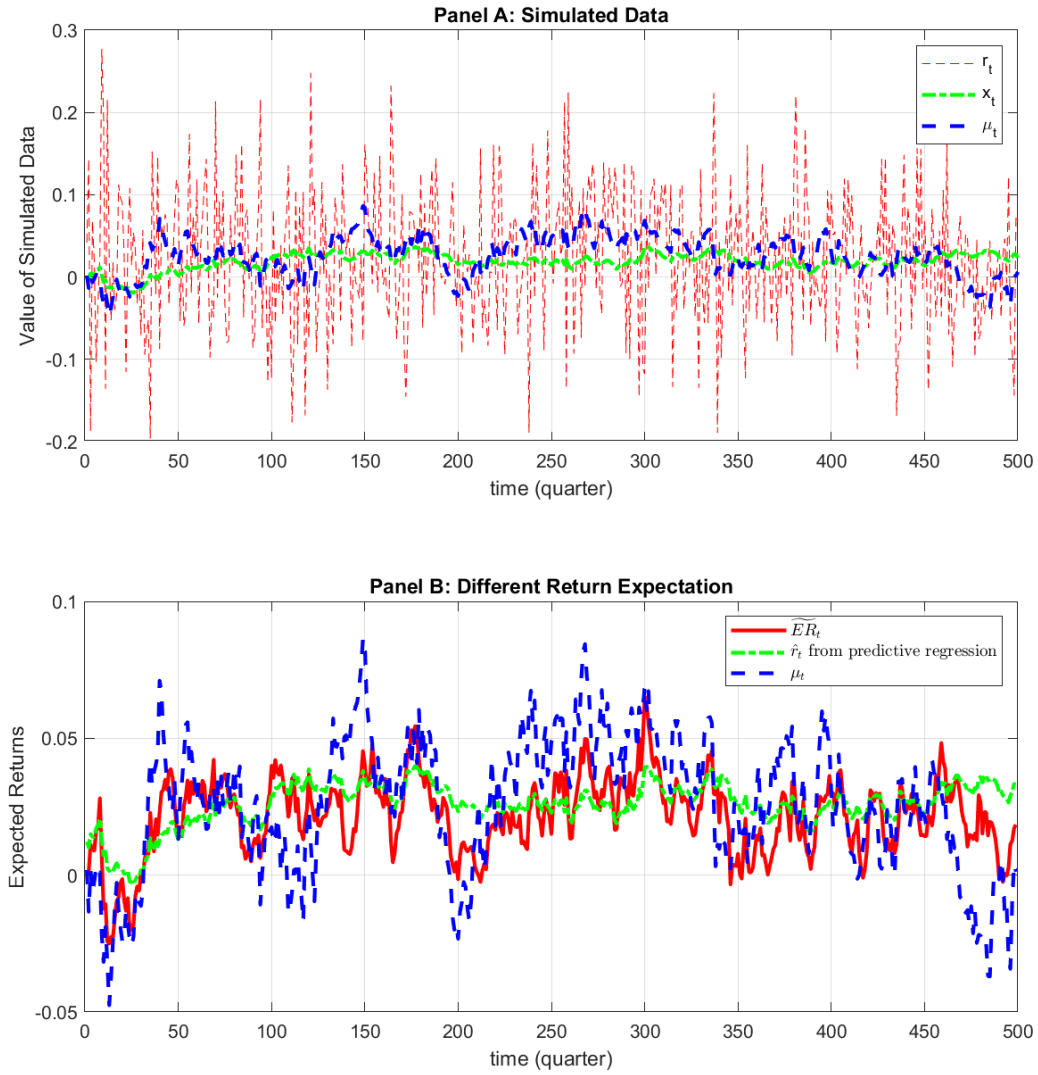
ratio against true expected returns and realized returns.

Panel B of Table 5 demonstrates the rational of using realized returns. In this sample, when combining realized returns with the observable predictor through the Kalman filter, the projected return out of sample is a much more accurate estimate for true expected returns compared to the simple predictive regression using  $x_t$ . Of course, this result is based on the correct prior beliefs on the covariance structure. In fact, this technique of using the Kalman filter to improve return forecasts is empirically verified in [Pástor and Stambaugh \(2009\)](#); [Van Binsbergen and Koijen \(2010\)](#).

Figure 2 shows subjective return expectations generated from simulated data. These are annual series from the 150th quarter onward in the simulation.  $\widetilde{ER}_t^{DR}$  denotes the (annualized) return expectation formed based on the prior that  $\widetilde{W}_t^\mu = 0.9687$ ,  $\widetilde{\beta}=0.9$  and  $\widetilde{E}_r = 0.03$ , where  $\widetilde{W}_t^\mu = \frac{\kappa_\mu \sigma_\mu}{\sigma_{v,t}}$  is defined in Section 2.3.2;  $\widetilde{ER}_t^{CF}$  denotes the (annualized) return expectation data based on the prior that  $\widetilde{W}_t^\mu = 0.01$ ,  $\widetilde{\beta}=0.96$ , and  $\widetilde{E}_r = 0.02$ . The other parameters in these two expectation series are calibrated to match moments of actual historical data of dividend yield and realized returns.

Figure 2 shows the model can match closely the heterogeneous return expectations graph 1 observed in the data, for a set of selected parameter values. As shown in the figure, even though the underlying data are the same, two forecasters can reach very different return expectations through the expectation formation model of 8, because of their different prior beliefs on how cash flows and the discount rate interact and because of their prior beliefs on how important the cash flow process is in driving asset prices.

Figure 5: Returns, Predictors and Return Expectation in Simulated Data



**Note:** In Panel A, 500 quarters of data are simulated based on the system in 3 to 7. Observable parameters are calibrated based on actual data, actual quarterly data on returns, and dividend yield:  $a = 0.97$ ,  $Er = 0.02$ ,  $Ex = 0.029$ ,  $\sigma_r = 0.08$ ,  $\rho_{rv} = \rho_{uv} = -0.89$ . Additional parameters are chosen based on results from Pástor and Stambaugh (2009):  $\beta = 0.9$ ,  $\sigma_u = 0.78$ ,  $\sigma_w = 0.0078$ ,  $\rho_{uw} = -0.71$  and  $\rho_{vw} = 0.5198$ .

## Appendix E Deriving Subjective Return Expectation Dynamics

I derive the expectation formation dynamics described in Equation (8). I further provide expressions for the steady-state parameters used in Section 2.3.

Investors infer the value of the unobservable expected returns  $\mu_t$  based on information set  $\mathcal{F}_t$  they observe from time 1 through time  $t$

$$\mathcal{F}_t = (z_1, z_2, \dots, z_t)$$

$$z_t = (r_t, x_t')'$$

Their prior belief, based on  $\mathcal{F}_0$ , is that the shocks follow multivariate normal:

$$\begin{bmatrix} u_{t+1} \\ v_{t+1} \\ \epsilon_{\mu,t+1} \end{bmatrix} | \mathcal{F}_0 \sim N \left[ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{uu} & \sigma_{uv} & \sigma_{u\mu} \\ \sigma_{uv} & \sigma_{vv} & \sigma_{v\mu} \\ \sigma_{u\mu} & \sigma_{v\mu} & \sigma_{\mu\mu} \end{bmatrix} \right] \quad (23)$$

where  $u_t$  and  $v_t$  are observable shocks to the returns and predictor vector  $x_t$ , respectively.  $\epsilon_{\mu,t}$  is a shock to the unobservable expected return process defined in Equation (5).<sup>37</sup> Since they also believe that the dynamics of expected return predictors follow (5) and (6), respectively, they consistently believe that

$$\begin{bmatrix} r_{t+1} \\ x_{t+1} \\ \mu_{t+1} \end{bmatrix} | \mathcal{F}_0 \sim N \left[ \begin{bmatrix} E_r \\ E_x \\ E_r \end{bmatrix}, \begin{bmatrix} V_{rr} & V_{rx} & V_{r\mu} \\ V_{rx} & V_{xx} & V_{x\mu} \\ V_{r\mu} & V_{x\mu} & V_{\mu\mu} \end{bmatrix} \right] \quad (24)$$

where the parameters in the variance-covariance matrix is a function of the parameters in (23) and the persistent parameters.

The investors use the Kalman filter to form their expectations. I denote

$$a_t = \tilde{E}(\mu_t | \mathcal{F}_{t-1}) \quad b_t = \tilde{E}(\mu_t | \mathcal{F}_t) \quad f_t = \tilde{E}(z_t | \mathcal{F}_{t-1})$$

<sup>37</sup>Notice here the shocks are following multivariate normal but the parameter values governing the variance-covariance matrix already are subject to investors' own prior beliefs.

as the conditional subjective expectations based on different information set. Let

$$\begin{aligned} P_t &= \widetilde{Var}(\mu_t | \mathcal{F}_{t-1}), & Q_t &= \widetilde{Var}(\mu_t | \mathcal{F}_t), & R_t &= \widehat{Var}(z_t | \mu_t, D_{t-1}) \\ S_t &= \widehat{Var}(z_t | \mathcal{F}_{t-1}), & G_t &= \widehat{Cov}(z, \mu_t | \mathcal{F}_{t-1}) \end{aligned}$$

be the subjective conditional variances that investors obtain after observing data.

Applying the updating algorithm of the Kalman filter,<sup>38</sup> we have

$$P_t = \beta^2 Q_{t-1} + \sigma_{\mu\mu} \quad (25)$$

$$S_t = \begin{pmatrix} Q_{t-1} + \sigma_{uu} & \sigma_{uv} \\ \sigma_{uv} & \sigma_{vv} \end{pmatrix} \quad (26)$$

$$G_t = \begin{pmatrix} \beta Q_{t-1} + \sigma_{u\mu} \\ \sigma_{v\mu} \end{pmatrix} \quad (27)$$

$$R_t = S_t - G_t P_t^{-1} G_t' \quad (28)$$

$$Q_t = P_t (P_t + G_t' R_t^{-1} G_t)^{-1} P_t \quad (29)$$

$$a_t = (1 - \beta) E_r + \beta b_{t-1} \quad (30)$$

$$f_t = \begin{pmatrix} b_{t-1} \\ (I - A) E_x + A x_{t-1} \end{pmatrix} \quad (31)$$

So the updated return expectation is

$$\begin{aligned} b_t &= a_t + G_t' S_t^{-1} (z_t - f_t) \\ &= a_t + m_t (r_t - b_{t-1}) + n_t' \left[ x_t - \widetilde{E}_t(x_t | D_{t-1}) \right] \\ &= a_t + m_t u_t + n_t' v_t \end{aligned} \quad (32)$$

and we arrive at Equation (8), since

$$\widetilde{E} R_{t|t} := b_t$$

---

<sup>38</sup>The internet appendix of [Pástor and Stambaugh \(2009\)](#) provides a similar derivation. See [Durbin and Koopman \(2012\)](#) for a more general treatment of Kalman filters and the state-space model in general.

and

$$\widetilde{ER}_{t|t-1} := a_t.$$

The parameters that govern the dynamics of  $m_t$  and  $n_t$  are dependent on the subjective prior beliefs of parameter values in (24). To see this, understand that the Kalman filtering process from Equation (25) to (32) are recursive relations starting from  $t = 2$ , which depend on values of parameters in  $t = 1$  when investors need to start from

$$\begin{aligned} a_1 &= E_r \\ P_1 &= V_{\mu\mu} \\ f_1 &= E_z \\ S_1 &= V_{zz} \\ G_1 &= V_{z\mu} \\ R_1 &= S_1 - G_1 P_1^{-1} G_1' \\ Q_1 &= P_1 (P_1 + G_1' R_1^{-1} G_1)^{-1} P_1 \\ b_1 &= a_1 + G_1' S_1^{-1} (z_1 - f_1) \end{aligned}$$

These values are based on the prior belief parameters and as analyzed in Section 2.3.1, not all of the parameters are identifiable through historical data, leaving room for heterogeneous expectation dynamics.

We know that

$$\begin{pmatrix} m_t & n_t' \end{pmatrix} = Cov(z_t', \mu_t | \mathcal{F}_{t-1}) Var(z_t | \mathcal{F}_{t-1})^{-1}$$

which is a function of  $Q_t$  defined in (29). The steady-state value of  $Q_t$  from (29) is

$$\begin{aligned} Q &= \frac{\sqrt{\lambda_1^2 - 4\lambda_2} - \lambda_1}{2} \\ \lambda_1 &= (1 - \beta^2) Var(u|v) + 2\beta Cov(u, \epsilon_\mu | v) - Var(\epsilon_\mu | v) \\ \lambda_2 &= Cov(u, \epsilon_\mu | v)^2 - Var(u|v) Var(\epsilon_\mu | v) \end{aligned}$$

so the steady-state values of the  $m_t$  and  $n_t$  are

$$m = [\beta Q + \text{Cov}(u, \mu|v)] [Q + \text{Var}(u|v)]^{-1} \quad (33)$$

$$n = (\sigma_{\mu v} - m\sigma_{uv})\sigma_{vv}^{-1} \quad (34)$$

## Appendix F Understanding What Drives Differences in Return Expectations

I provide a more technical analysis to support the intuitive explanations in Section 2.3.2. Furthermore, I provide more details about how I make the plot of Figure 3.

The two prior beliefs can lead to either contrarian or extrapolative return expectations. To understand how, note that return expectations defined in (8) in the steady-state depend on the past returns through

$$\tilde{m} = [\beta Q + \text{Cov}(u_t, \epsilon_{\mu,t}|v)] / [Q + \text{Var}(u_t|v_t)]$$

where  $Q$  is the steady-state variance of  $\tilde{E}(r_t|\mathcal{F}_t)$ .<sup>39</sup> An investor will only appear to be contrarian if and only if  $\tilde{m} < 0$  or

$$\text{Cov}(u_t, \epsilon_{\mu,t}|v_t) < -\beta Q$$

This condition is equivalent to

$$\rho_{d\mu} < -\frac{\beta Q}{\sigma_d \sigma_\mu} + \frac{\sigma_v - \rho_{uv} \sigma_u}{\sigma_d} \rho_{v\mu} \quad (35)$$

$$\approx -\frac{\beta Q}{\sigma_d \sigma_\mu} + 1.58 \rho_{v\mu} \quad (36)$$

where the approximation (36) is due to the fact that investors will have fairly accurate estimates from the data about  $(\sigma_v - \rho_{uv} \sigma_u) / \sigma_d$ .<sup>40</sup>

This condition (36) shows that whether an investor appears contrarian depends largely on the value of  $\rho_{v\mu}$ : if  $\rho_{v\mu}$  is very large and positive, investors will likely be contrarian because a

<sup>39</sup>The expression of  $Q$  is given in Appendix E

<sup>40</sup>This is because  $\rho_{uv}$  is easy to measure empirically so we have  $\rho_{uv} \approx \rho_{r,dp} = -0.89$ , which leads to  $(\sigma_v - \rho_{uv} \sigma_u) / \sigma_d = 1.58$  and the last approximation in Equation (36) therefore follows.

large set of values of  $\rho_{d\mu}$  would lead to a negative  $\tilde{m}$ .

Furthermore, based on the present value ratio in Equation (13),  $\rho_{v\mu}$  is given by:<sup>41</sup>

$$\rho_{v\mu} = \frac{\kappa_\mu \sigma_\mu}{\sigma_v} - \rho_{g\mu} \frac{\kappa_g \sigma_g}{\sigma_v} \quad (37)$$

where  $\sigma_v$  is the volatility of the dividend-price ratio. This condition shows the value of  $\rho_{v\mu}$  depends on (i)  $W_\mu := \frac{\kappa_\mu \sigma_\mu}{\sigma_v}$ , or how investors interpret the importance of discount rate news and (ii)  $\rho_{\mu,g}$ : how they interpret expected cash flow news for future returns.

Notice that  $W_\mu := \frac{\kappa_\mu \sigma_\mu}{\sigma_v}$  does not exactly equal the conventional variance decomposition of dividend-price ratio, or

$$\begin{aligned} V_{\mu,dp} &= \frac{Var(\mu_t)}{Var(dp_t)} \\ &= \frac{1 - \phi^2}{1 - \beta^2} \frac{1}{\Delta + \frac{1}{W_\mu^2}} \end{aligned} \quad (38)$$

where

$$\Delta := \frac{\kappa_\mu^2 (\beta - \phi)^2}{1 - \beta^2}$$

The Equation (38) is obtained by taking unconditional variance on Equation (14) and (5) for the denominator and numerator, respectively.

Equation (38) shows that although  $W_\mu$  and the  $V_{\mu,dp}$  are not the same,  $W_\mu$  does increase with  $V_{\mu,dp}$  if the persistent parameters are held to be constant.

## Appendix G Estimating Expectation Formation Process

I estimate the system from (15) to (19) in three steps. First, I estimate the shocks  $\epsilon_{\mu,t}$ ,  $\epsilon_{g,t}$ , and  $\epsilon_{d,t}$  as well as parameters in the predictive system as captured in equations (15), (16), and (17) together with the parameters in Equation (2). To estimate this, I write the system into a state-space form and estimate the parameters using the Kalman filter based on the maximum-likelihood function. I illustrate the state-space form of the system in Section G.1. Second, the parameters in

---

<sup>41</sup>Multiply both sides of the equation and taking expectation

Equation (18) are estimated separately by ordinary least squares together with the residuals. Finally, the correlations between innovations in predictors  $x_t$  and  $\epsilon_{\mu,t}$ ,  $\epsilon_{g,t}$  and  $\epsilon_{d,t}$  are using these estimated series.<sup>42</sup>

### G.1 A Simplified System With Return Expectations

$$\hat{r}_{t+1} = \frac{1}{L(\beta)} \hat{\mu}_{t+1}^A + \epsilon_{\Delta d,t+1} - \rho \kappa_{\mu}(\beta) \epsilon_{\mu,t+1} + \rho \kappa_g(\phi_g) \epsilon_{g,t+1} \quad (39)$$

$$\hat{d}p_{t+1} = \phi_g \hat{d}p_t + \frac{\kappa_{\mu}(\beta - \phi_g)}{L(\beta)} \hat{\mu}_{t+1}^A + \kappa_{\mu} \epsilon_{\mu,t+1} - \kappa_g \epsilon_{g,t+1} \quad (40)$$

$$\hat{\mu}_{t+1}^A = \beta \hat{\mu}_t^A + L(\beta) \epsilon_{\mu,t+1} \quad (41)$$

where

$$\begin{aligned} L(\beta) &= \sum_{k=0}^3 \beta^k \\ \kappa_{\mu}(\beta) &= \frac{1}{1 - \rho\beta} \\ \kappa_g(\phi_g) &= \frac{1}{1 - \rho\phi_g} \\ B_{dp,\mu}(\beta, \phi_g) &= \kappa_{\mu}(\beta - \phi_g) \end{aligned}$$

and

$$\begin{pmatrix} \epsilon_{\Delta d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix} \sim N \left[ 0, \begin{pmatrix} \sigma_d^2 & \sigma_{\mu d} & \sigma_{gd} \\ \sigma_{\mu d} & \sigma_{\mu}^2 & \sigma_{\mu g} \\ \sigma_{gd} & \sigma_{\mu g} & \sigma_g^2 \end{pmatrix} \right] \quad (42)$$

This system from Equation (39) to (42) is a linear system of underlying shocks and is non-linear with respect to parameters

$$\theta = (\phi_g, \beta, \sigma_d, \sigma_{\mu}, \sigma_g, \sigma_{\mu d}, \sigma_{gd}, \sigma_{\mu g})'$$

Given the normality assumption in (42), the system can be estimated by maximum-likelihood

<sup>42</sup>These estimates are consistent estimates of the parameters. Potentially, I can use these estimates to re-estimate the entire system all together using maximum-likelihood. The resulting estimates are similar to the three-step approach estimated here. Details of the estimation is provided in G.

estimation. I write the system into state-space form.

The observable vector is

$$y_{t+1} = \begin{pmatrix} \hat{r}_{t+1} \\ \hat{d}p_{t+1} \\ \hat{\mu}_{t+1}^A \end{pmatrix}$$

and the latent processes

$$\alpha_t = \begin{pmatrix} \mu_{t+1} \\ \mu_t \\ \hat{d}p_{t+1} \\ \hat{d}p_t \\ \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix}$$

The dynamics of the measurement equations are captured by

$$\begin{pmatrix} \hat{r}_{t+1} \\ \hat{d}p_{t+1} \\ \hat{\mu}_{t+1}^A \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{L(\beta)} & 0 & 0 & 1 & -\rho\kappa_\mu & \rho\kappa_g \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mu_{t+1} \\ \mu_t \\ \hat{d}p_{t+1} \\ \hat{d}p_t \\ \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 0 & \frac{1}{L(\beta)} & 0 & 0 & 1 & -\rho\kappa_\mu & \rho\kappa_g \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$d = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\eta_t = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$H = 0_{3 \times 3}$$

and the state-equation is characterized by

$$\begin{pmatrix} \mu_{t+1} \\ \mu_t \\ \hat{d}p_{t+1} \\ \hat{d}p_t \\ \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{B_{dp,\mu}(\beta,\phi_g)}{L(\beta)} & 0 & \phi_g & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{\mu}_t^A \\ \hat{\mu}_{t-1}^A \\ dp_t \\ dp_{t-1} \\ \epsilon_{d,t} \\ \epsilon_{\mu,t} \\ \epsilon_{g,t} \end{pmatrix} + \begin{pmatrix} 0 & L(\beta) & 0 \\ 0 & 0 & 0 \\ 0 & \kappa_\mu(\beta) & -\kappa_g(\phi_g) \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix}$$

$$T = \begin{pmatrix} \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{B_{dp,\mu}(\beta,\phi_g)}{L(\beta)} & 0 & \phi_g & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$c = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$\epsilon_t = \begin{pmatrix} \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix}$$

$$R = \begin{pmatrix} 0 & L(\beta) & 0 \\ 0 & 0 & 0 \\ 0 & \kappa_\mu(\beta) & -\kappa_g(\phi_g) \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q = \begin{pmatrix} \sigma_d^2 & \sigma_{\mu d} & \sigma_{gd} \\ \sigma_{\mu d} & \sigma_\mu^2 & \sigma_{\mu g} \\ \sigma_{gd} & \sigma_{\mu g} & \sigma_g^2 \end{pmatrix}$$

where  $\Sigma_x$  has been pre-estimated using the OLS residuals. The system has an initial state vector

$$\alpha_{1|0} = (I_7 - T)^{-1}c$$

$$vec(P_{1|0}) = (I_{49} - T \otimes T)^{-1}vec(RQR')$$

## Appendix H Consistency in Analysts' Return and Cash Flow Expectations

In the estimation framework presented in the previous section, the cash flow expectation is backed out through the present value relation based on return expectation and valuation ratio similar to the VAR framework developed by [Campbell \(1991\)](#). Inherently, this framework assumes that subjective return and cash flow expectations of market participants are consistent with the present value model and the mean-reverting expectation dynamics. To see if analysts' own expectations are indeed consistent, I compare analysts' own cash flow expectations, which is observable, with the implied cash flow expectations based on return expectations.

In summary, I find that directly observed analysts' cash flow expectations are broadly consis-

tent with the implied cash flow expectations based on return expectations and price-fundamental ratios, although there are finer nuances in the observed expectation dynamics directly reported by analysts.

Table 14 shows the correlations between various directly observed cash flow expectations (shocks) with implied expectations (shocks). First, implied cash flow expectations (shocks) are strongly, although imperfectly correlated with analysts' own cash flow expectations (shocks). In particular, as shown in Table 14a and 14b, the implied dividend growth expectations and analysts' earnings growth expectations are 82% correlated and the shocks are about 50% correlated. This imperfect correlation could potentially be due to three reasons: first, there are measurement errors on the expectations series; second, the expectations on returns or cash flows are more complicated than a simple AR(1) process, for example, it contains a term structure of cash flow expectations; third, analysts' expectations processes are not perfectly consistent. As a result of the imperfect correlation, the correlation between the shocks on return expectations and cash flow expectations are less strong than estimated through the model, as shown in Table 14b. Finally, the multi-variable regression results in Table 14c show that the implied cash flow expectations are correlated by more than one observed cash flow expectation measure, including earnings and dividend expectations.

**Table 14: Correlations Between Implied Dividend Growth Expectation and Analysts' Cash Flow Expectations**

This table compares analysts' reported cash flow expectations with those implied by their return expectations.  $g$  is the implied cash flow expectation from the model estimation. *Analyst.Div.Growth*, *Analyst.E.Growth*, *Analyst.Payout*, and *LTG* are analysts' reported estimates of dividend growth, earnings growth, dividend payout, and long-term growth, respectively. *Actual.Div.Growth* is actual dividend growth. *Implied.shock.ER*, *implied.shock.g*, and *implied.shock.d* are analysts' expectations of shocks to return expectations, cash flows, and dividends, respectively, as implied by the analysts' return expectations based on the expectation formation framework proposed here. *Shock.analyst.earnings*, *shock.analyst.div.growth*, and *shock.analyst.LTG* are the analysts' reported predictions of shocks to earnings, dividend growth, and long-term growth, respectively. All statistics are calculated from the sample between 2003:Q1 to 2019:Q4, where all series have data available. \* $p < 0.1$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$ .

(a) Pair-wise correlations between implied and directly reported cash flow expectations

|                           | $g$     | <i>Analyst.Div.Growth</i> | <i>Analyst.E.Growth</i> | <i>Analyst.Payout</i> | <i>LTG</i> | <i>Actual.Div.Growth</i> |
|---------------------------|---------|---------------------------|-------------------------|-----------------------|------------|--------------------------|
| $g$                       | 1.00    |                           |                         |                       |            |                          |
| <i>Analyst.Div.Growth</i> | 0.79*** | 1.00                      |                         |                       |            |                          |
| <i>Analyst.E.Growth</i>   | 0.82*** | 0.81***                   | 1.00                    |                       |            |                          |
| <i>Analyst.Payout</i>     | -0.03   | -0.06                     | -0.20                   | 1.00                  |            |                          |
| <i>LTG</i>                | 0.51*** | 0.46***                   | 0.54***                 | 0.11                  | 1.00       |                          |
| <i>Actual.Div.Growth</i>  | 0.67*** | 0.82***                   | 0.73***                 | -0.30**               | 0.37***    | 1.00                     |

(b) Pair-wise correlations between implied and directly reported cash flow expectation shocks

|                                 | <i>implied.shock.ER</i> | <i>implied.shock.g</i> | <i>implied.shock.d</i> | <i>shock.analyst.earnings</i> | <i>shock.analyst.div.growth</i> | <i>shock.analyst.LTG</i> |
|---------------------------------|-------------------------|------------------------|------------------------|-------------------------------|---------------------------------|--------------------------|
| <i>implied.shock.ER</i>         | 1.00                    |                        |                        |                               |                                 |                          |
| <i>implied.shock.g</i>          | -0.54***                | 1.00                   |                        |                               |                                 |                          |
| <i>implied.shock.d</i>          | -0.38***                | -0.11                  | 1.00                   |                               |                                 |                          |
| <i>shock.analyst.earnings</i>   | -0.04                   | 0.50***                | -0.24*                 | 1.00                          |                                 |                          |
| <i>shock.analyst.div.growth</i> | -0.15                   | 0.47***                | -0.06                  | 0.80***                       | 1.00                            |                          |
| <i>shock.analyst.LTG</i>        | -0.19                   | 0.36***                | 0.00                   | 0.51***                       | 0.51***                         | 1.00                     |

(c) Linear Regressions With Multiple Regressors

|                           | Dependent variable:     |                        |                        |
|---------------------------|-------------------------|------------------------|------------------------|
|                           | $g$                     |                        |                        |
|                           | (1)                     | (2)                    | (3)                    |
| <i>Analyst.Div.Growth</i> | 0.618***<br>(0.079)     |                        | 0.268**<br>(0.132)     |
| <i>Analyst.E.Growth</i>   |                         | 0.738***<br>(0.158)    | 0.479***<br>(0.125)    |
| <i>Analyst.Payout</i>     |                         | 0.312<br>(0.195)       | 0.223<br>(0.216)       |
| <i>LTG</i>                |                         | 0.496<br>(1.852)       | 0.477<br>(1.398)       |
| Constant                  | -0.073***<br>(0.025)    | -0.252<br>(0.190)      | -0.219<br>(0.190)      |
| Observations              | 62                      | 62                     | 62                     |
| R <sup>2</sup>            | 0.626                   | 0.689                  | 0.728                  |
| Adjusted R <sup>2</sup>   | 0.619                   | 0.673                  | 0.708                  |
| Residual Std. Error       | 0.076 (df = 60)         | 0.070 (df = 58)        | 0.066 (df = 57)        |
| F Statistic               | 100.284*** (df = 1; 60) | 42.768*** (df = 3; 58) | 38.058*** (df = 4; 57) |

Table 15a shows the variance decomposition based on directly used cash flow and return expectations measured by regressing the expectation measures on the dividend-price ratio. The coefficients on the dividend-price ratio from the regression can be interpreted directly as the variance explained by the expectation measure. Despite the implied cash flow growth expectations being imperfectly correlated with a particular observed cash flow expectation, the results show that cash flow expectations alone explain 99% of the price-dividend ratio variation, while return expectations only explain about 10%. These results are consistent with the findings presented in the previous section, confirming that the model is broadly consistent with the data. The small and insignificant coefficient on an analyst's long-term growth expectations is also interesting, which means that most of the variation of price-dividend ratios, from the analyst's own perspective, is due to short-term cash flow expectations, consistent with the findings of De la O and Myers (2020). However, in their framework, De la O and Myers (2020) use the CFO expectations as the measure for return expectations. As shown in the right-most column of Table 15a, such an assumption can result in different conclusions regarding how return expectations are related to prices and cash flow expectations, which is demonstrated by the different signs of the regression coefficients between the analyst and the CFO return expectations.

Finally, Table 15b shows that the variance decomposition using the subjective expectations based directly on observed cash flow expectations is very different from the variance decomposition using the VAR framework based on the dividend-price ratio employed in Cochrane (2011). The implied long-run coefficient on  $\log(D/P)$  in the return equation from this framework is 0.678, which means that based on the same sample period, an econometrician would conclude that the discount rate variation is the main driver behind the variation of price-dividend ratios, instead of short-term cash flow growth. These results highlight the difference between the subjective expectations and the objective expectations, when making inferences on the variance decomposition of fundamental-price ratios.

Overall, the results in this section show that analysts' expectations about future returns and cash flows are broadly consistent with the simple present value model described above. However, the reported expectation data might have nuances that are not captured by the simple mean-reverting process, and this issue is worthy of further research. Potentially, there could be a term structure on both return expectations and cash flow expectations that is not considered

in the current model. This is outside of the scope of the current paper so I leave it to future researchers.

Table 15: Variance Decomposition of Log Dividend-Price Ratios

This table reports the variance decomposition of the (log) dividend-price ratio based on analysts' subjective return expectations (panel (a)) and the VAR as used in [Cochrane \(2011\)](#) (panel (b)). The subjective variance decomposition is computed by regressing subjective expectations directly on (log) dividend yield, as in [De la O and Myers \(2020\)](#). *Div.Growth.1y* is the analysts' cash flow expectations, *LTG* is the analysts' long-term growth expectations, *ER.Analyst* are value-weighted sell-side analyst return expectations for the S&P 500 index, and *ER.CFO* are CFOs' year-over-year return expectations from the Duke University survey. *Next.Year.Excess.Ret* is the actual next year excess returns, and *Next.Qtr.Log(DP)* is the actual next quarter log dividend-price ratio. \* $p < 0.1$ ; \*\* $p < 0.05$ ; \*\*\* $p < 0.01$ .

## (a) Variance Decomposition Subjective Expectations Analysts

|                         | <i>Dependent variable:</i> |                  |                     |                      |
|-------------------------|----------------------------|------------------|---------------------|----------------------|
|                         | <i>Div.Growth.1y</i>       | <i>LTG</i>       | <i>ER.Analyst</i>   | <i>ER.CFO</i>        |
|                         | (1)                        | (2)              | (3)                 | (4)                  |
| log(DP)                 | 0.992***<br>(0.214)        | 0.049<br>(0.031) | 0.098***<br>(0.032) | −0.063***<br>(0.021) |
| Adjusted R <sup>2</sup> | 0.615                      | 0.209            | 0.183               | 0.403                |

(b) Variance Decomposition VAR as in [Cochrane \(2011\)](#)

|                              | <i>Dependent variable:</i>  |                         |
|------------------------------|-----------------------------|-------------------------|
|                              | <i>Next.Year.Excess.Ret</i> | <i>Next.Qtr.Log(DP)</i> |
|                              | (1)                         | (2)                     |
| Log(DP)                      | 0.362***<br>(0.128)         | 0.839***<br>(0.101)     |
| Implied Long-Run Coefficient | 0.678<br>65                 |                         |
| Adjusted R <sup>2</sup>      | 0.088                       | 0.700                   |

## **I Internet Appendix for “Subjective Return Expectations”**

## I.1 Additional Summary Statistics

The data set on analyst target prices has good and stable coverage for a large number of firms, especially when compared to surveys from CFOs and others that were studied in the literature. The coverage for the S&P 500 is significantly better than that of smaller firms, which is the reason why I choose the S&P 500 universe as the venue for most of the empirical tests. Table 16 shows the summary statistics for the variables used in this paper.

Panels (a) and (b) show the coverage of return expectations for the S&P 500 firms and all other firms. The number of analysts who completed the survey far exceeds those of CFOs (from Duke University), or retail investors (from Shiller individuals), which have 390 and 81 respondents, respectively.<sup>43</sup> At a point in time, there are about 2700 analysts from 236 brokerage firms in the universe, among which 1410 analysts from 144 firms at a point in time cover S&P 500 firms, or 2.6 analysts per firm. The coverage deteriorates as the firm size becomes smaller, as shown in Panel (b), where the number of analysts per firm reduces to only 0.71 for the entire COMPUSTAT universe. For this reason, I use the S&P 500 universe as the main data set for analysis. On the other hand, the median analyst in the data set covers 35 firms, with a standard deviation of 22 firms. This is consistent with the practice of one analyst typically covering a specific sector.

For S&P 500 firms, analysts revise their forecasts on average every 20 days, with a standard deviation of about 16 days. Notice that when constructing the sample, I exclude all estimates that are older than 60 days. In Internet Appendix I.2.1, I describe the timing of the issuance in more detail. The existing surveys on CFOs and retail investors are all in quarterly frequency.

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<sup>43</sup>These numbers are as reported in Table 1 of Adam et al. (2021).

Figure 6: Coverage statistics S&P 500 over time

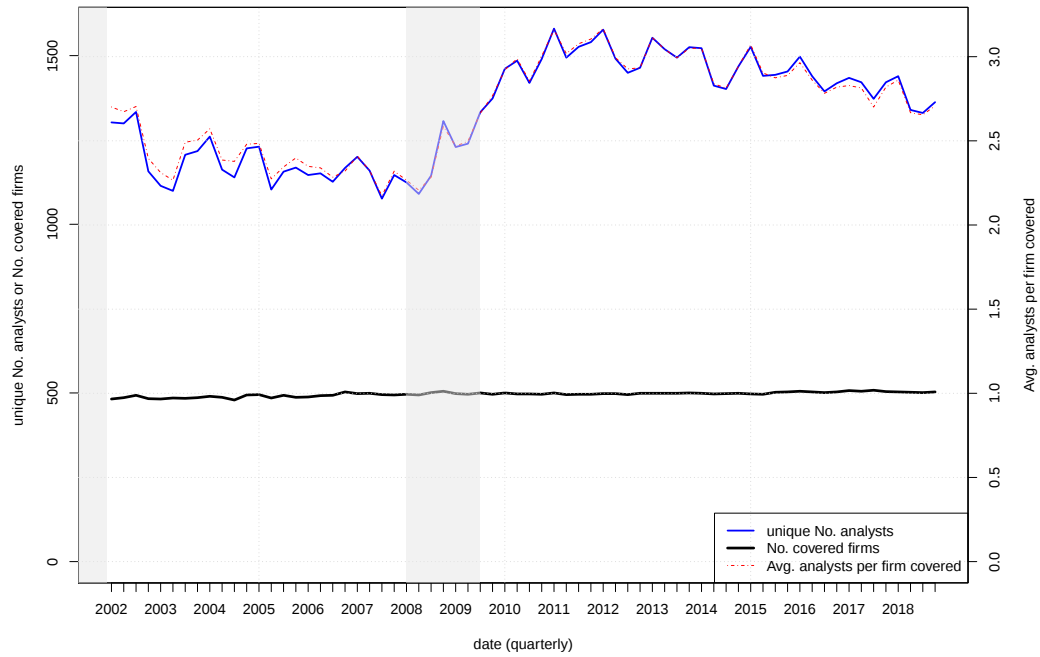


Figure 7: Coverage statistics all firms over time

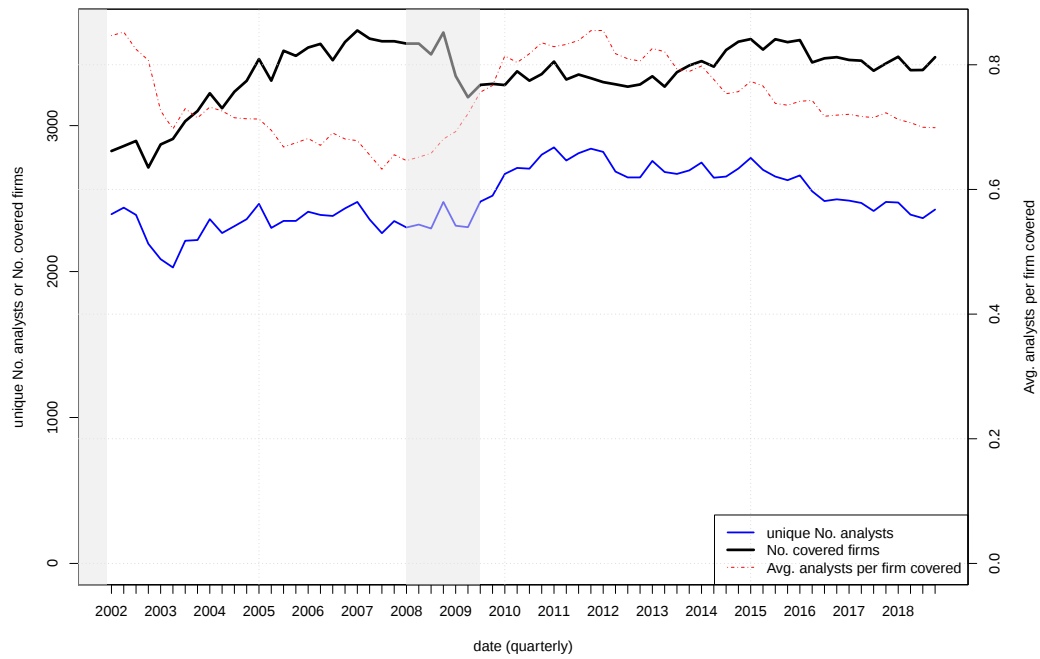


Figure 6 and Figure 7 show that the coverage of analyst return expectations data is stable over time for both the S&P 500 and the CRSP-COMPUSTAT universe, respectively. For the S&P 500 firms, the number of analysts submitting price targets goes from approximately 1200 before 2008 to about 1500 in the last decade. The average number of firms covered stays very close to 500 over time. Figure 7 shows coverage over time for all firms with analyst price targets, which shows stable coverage as well.

Table 16: Summary Statistics

This table provides additional summary statistics of the data set used for analysts' subjective return expectations. The sample period is from 2002-01-01 to 2018-12-31. Monthly and quarterly data are measured at the calendar month end and quarter end, respectively. *nr.brokerage.firms* is the number of firms in the data at each data point. *nr.analysts* is the number of analysts at each data point. *nr.covered.firms* is the number of firms with analyst estimates. *avg.days.since.last.revise* and *std.days.since.last.revise* is the average number and standard deviation, respectively, of days between updates of an analyst's forecast for a firm. *avg.nr.analysts.per.firm* is the average number of analysts covering each firm. *ER.analyst* are value-weighted sell-side analyst return expectations for the S&P 500 index. *fwd.12m.E/P* denotes forward 12-month earnings expectations, constructed using one- and two-fiscal year ahead earnings expectations, divided by the market cap. *LTG* are analyst long-term growth expectations. *ann.earnings* are annual actual earnings. *ROE* is actual annual earnings divided by total book value; *B/M* are book-to-market ratio.

## (a) Coverage Statistics S&amp;P 500

| Statistic                  | Mean      | St. Dev. | Min    | Pctl(25) | Pctl(75) | Max    |
|----------------------------|-----------|----------|--------|----------|----------|--------|
| nr.brokerage.firms         | 143.353   | 15.717   | 100    | 135.8    | 152      | 181    |
| nr.analysts                | 1,334.926 | 151.938  | 1,076  | 1,167.8  | 1,461.8  | 1,580  |
| nr.covered.firms           | 496.162   | 6.601    | 479    | 493.8    | 500.2    | 508    |
| avg.days.since.last.revise | 20.141    | 1.173    | 17.600 | 19.347   | 20.776   | 22.112 |
| std.days.since.last.revise | 16.374    | 0.852    | 14.633 | 15.697   | 16.953   | 18.422 |
| avg.nr.analysts.per.firm   | 2.689     | 0.289    | 2.174  | 2.398    | 2.929    | 3.167  |

## (b) Coverage Statistics All

| Statistic                  | Mean      | St. Dev. | Min    | Pctl(25) | Pctl(75) | Max    |
|----------------------------|-----------|----------|--------|----------|----------|--------|
| nr.brokerage.firms         | 230.338   | 27.148   | 125    | 228.8    | 241.2    | 271    |
| nr.analysts                | 2,495.632 | 195.426  | 2,028  | 2,354.5  | 2,669    | 2,851  |
| nr.covered.firms           | 3,360.691 | 209.747  | 2,713  | 3,282.8  | 3,514.2  | 3,652  |
| avg.days.since.last.revise | 20.017    | 1.033    | 17.523 | 19.393   | 20.481   | 23.032 |
| std.days.since.last.revise | 15.928    | 0.661    | 14.602 | 15.479   | 16.364   | 17.625 |
| avg.nr.analysts.per.firm   | 0.744     | 0.061    | 0.632  | 0.699    | 0.804    | 0.855  |

## (c) S&amp;P 500 Firm-level Analyst Expectation Data

| Statistic   | N       | Mean  | St. Dev. | Min    | Pctl(25) | Pctl(75) | Max   |
|-------------|---------|-------|----------|--------|----------|----------|-------|
| ER.analyst  | 101,985 | 0.135 | 0.110    | -0.107 | 0.066    | 0.185    | 0.549 |
| fwd.12m.E/P | 103,575 | 0.065 | 0.031    | -0.038 | 0.048    | 0.080    | 0.162 |
| LTG         | 99,094  | 0.115 | 0.080    | -0.114 | 0.074    | 0.149    | 0.462 |

## (d) Other S&amp;P 500 firm-level data

| Statistic             | N       | Mean   | St. Dev. | Min     | Pctl(25) | Pctl(75) | Max       |
|-----------------------|---------|--------|----------|---------|----------|----------|-----------|
| tot.ret               | 103,676 | 0.008  | 0.085    | -0.255  | -0.037   | 0.054    | 0.266     |
| mcap(in bil.)         | 103,693 | 27.927 | 51.410   | 0.083   | 6.570    | 26.041   | 1,099.436 |
| ann.earnings(in bil.) | 103,084 | 1.553  | 3.264    | -53.557 | 0.322    | 1.465    | 56.518    |
| ROE                   | 102,099 | 0.183  | 0.233    | -0.701  | 0.089    | 0.233    | 1.504     |
| B/M                   | 102,360 | 0.483  | 0.415    | -0.034  | 0.211    | 0.629    | 2.440     |

The distributional statistics in Panel (c) of Table 16 show that the average analyst return expectation for firms in the S&P 500 universe is 13.4%. This is much higher than the realized average total return of about 9.6% per year, as shown in Panel (d). The positive bias documented here is consistent with those in the previous literature, such as Brav and Lehavy (2003); Engelberg et al. (2019). Besides, the analyst earnings forecasts statistics are similar to those documented in Bordalo, Gennaioli, Porta, and Shleifer (2019); De la O and Myers (2020).

## **I.2 The Timing of Analyst's Price Target Issuance**

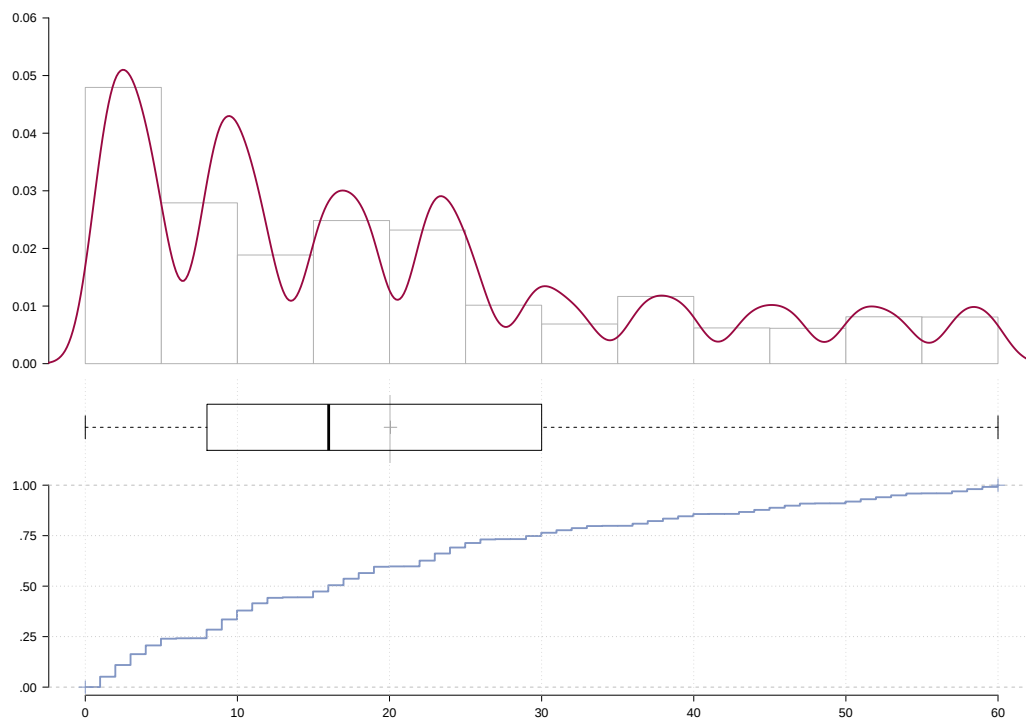
I describe in more detail the timing of analysts' issuance of price targets. First, I examine the frequency at which an analyst issues a price target. Second, I investigate whether analysts issue more price targets during firms' earnings announcement months. The results in this section show that the median analyst issues a new price target every 16 days. For analysts who issue price targets less frequently, they tend to issue new estimates during earnings announcement months, and particularly on or one day after the earnings announcement.

### **I.2.1 How Frequent an Analyst Issues a Price Targets**

On average, a median analyst issues a new price target every 16 days for a particular firm they cover. Only 2% of these estimates are the same as the price targets issued previously.

Figure 8 plots the empirical distribution of the number of days between an analyst's newly issued price target and their previous issuance on the same firm. On average, a median analyst issues a new price target every 16 days, with a mean of 20 days. Furthermore, about 75% of the analysts issue a new estimate each month, or within 30 days. Combining this information together with Figure 9, those analysts who issue less frequently, say those who issue a new estimate every 60 days, will typically issue during the earnings month each quarter.

Figure 8: Empirical Distribution (Upper: PDF, Lower: CDF) of the No. of days between two subsequent issuance of price targets by the same analyst for the same firm



**Note:** Upper panel: Probability density function; Lower panel: Cumulative distribution function.

Another question is, among these frequent updates, in how many incidences analysts would maintain, or issue the same price targets as previous price targets? Table 17 shows that only about 2% of all issued price targets are the same as the previous one. This percentage is much lower for return expectations. Over time, this percentage is also stable, varying at about 1% standard deviation per quarter.

Table 17: Summary Statistics: Percentage of Analysts Who Issues the Same Price Target Each Quarter

This table examines how often analysts maintain the same price targets from one forecast to the next. *prop.maintain.PTG* is calculated as follows. First, for each analyst that issued a price target for a particular firm, the previous issuance for the same firm was compared, if available. Subsequently, for each calendar quarter, count the number of incidences where the current issuance is equal to the previous quarter and the number that they are not equal. *prop.maintain.PTG* is the proportion of the former (same price target) divided by the total number of analyst issuance. *prop.maintain.ER* is the same proportion but calculated using expected returns, instead of price target estimates.

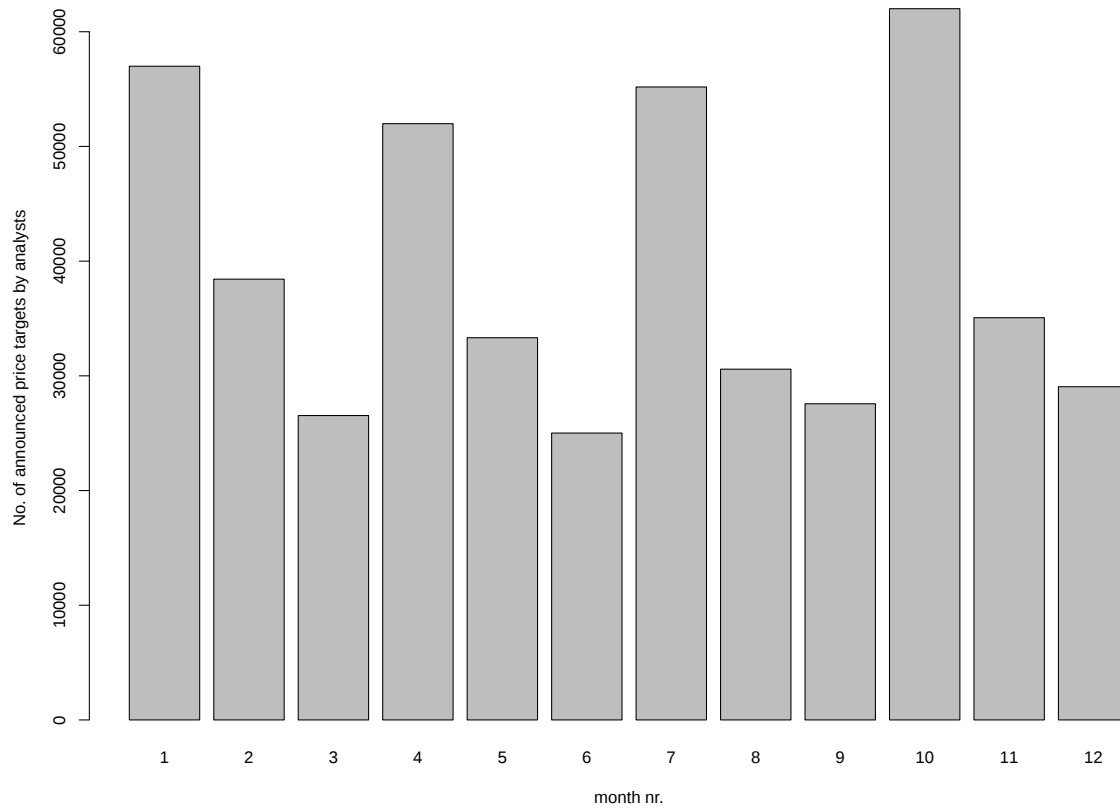
| Statistic                | N  | Mean   | St. Dev. | Min   | Pctl(25) | Pctl(75) | Max   |
|--------------------------|----|--------|----------|-------|----------|----------|-------|
| <i>prop.maintain.PTG</i> | 68 | 0.023  | 0.011    | 0.004 | 0.015    | 0.030    | 0.053 |
| <i>prop.maintain.ER</i>  | 68 | 0.0004 | 0.001    | 0.000 | 0.000    | 0.0002   | 0.011 |

### I.2.2 Price Targets Issuance and Firm Quarterly Earnings Announcements

Analysts are more likely to announce their price targets during the first month of each quarter, during which more firms announce their quarterly earnings. During firms' announcement months, analysts typically announce new price targets on or shortly following the announcement day.

Figure 9 plots the number of announced price targets by all analysts in the sample of S&P 500 firms for the whole sample. The total price targets announced during Jan., Apr. Jul. and Oct. are about 48% of all announced, higher than the 33% if they are announced evenly through out the year. This is similar to the seasonal pattern of earnings announcements.

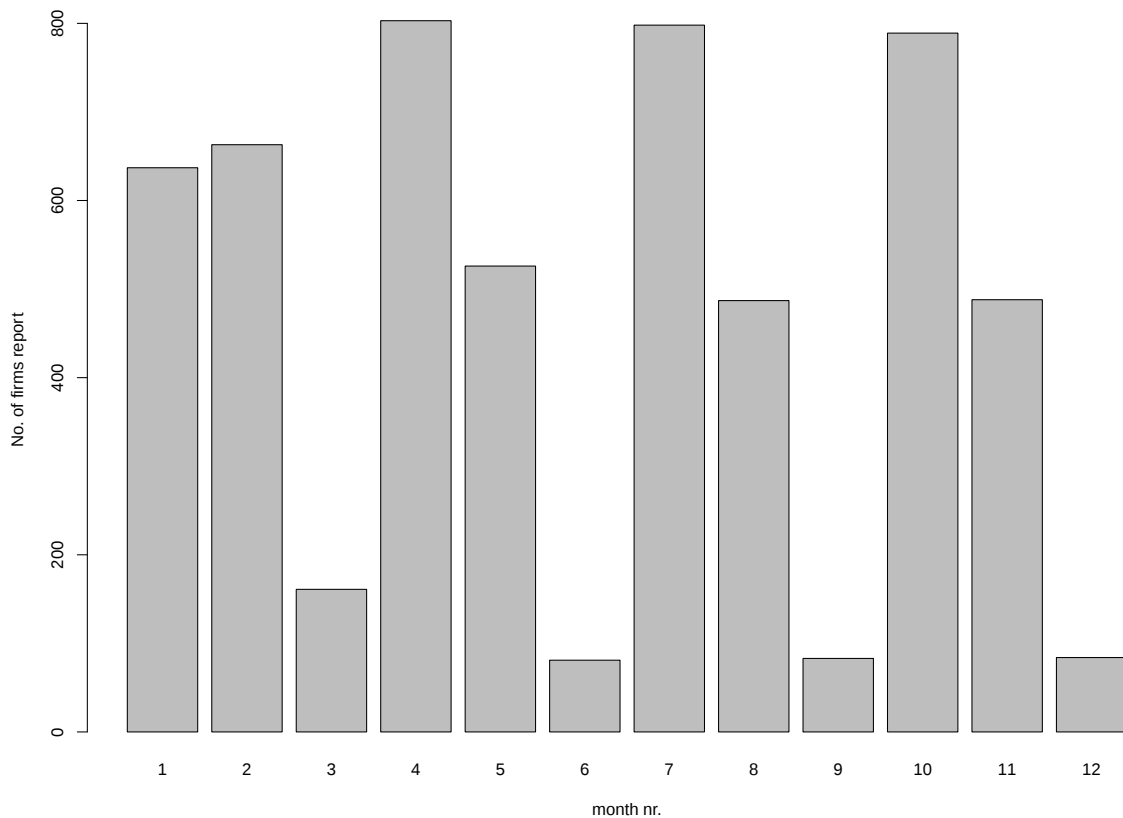
Figure 9: Number of Price Targets Announced by Month, S&P 500 Universe



**Note:** Sample starts from 2002-01-01 to 2018-12-31. The No. of announced price targets by analysts count each single analyst submitted price targets for a particular firm during a month as one. Price targets that are issued longer than 60 days ago are excluded.

In fact, the seasonal pattern of firms' earnings announcements are much more dramatic compared to the announcements of analysts' price targets. As shown in Figure 10, the number of earnings announcements are almost eight times than those in Q2, Q3 and Q4. This suggests that analysts' price targets changes are not only driven by firms' earnings.

Figure 10: Number of Earnings Announcements By Month, S&P 500 Universe



**Note:** Sample starts from 2002-01-01 to 2018-12-31. The No. of firms that report earnings count all firms that ever are in the S&P 500 index throughout the sample period and therefore can exceed 500.

During the month that firms do announce, about 50% of all new price target issuance is concentrated on or one day after the earnings day. The distribution of the number of days between announcement of price targets and the earnings announcement day is summarized in Table 18.

Table 18: Summary Distribution: Days Between Announcement of Price Targets and Nearest Earnings Announcements Within Each Month

This table reports length of time following a firm's earnings announcement that analysts' update their forecasts. *days.after.earnings* is that time period measured in days.

| Statistic           | N       | Mean   | Median | St. Dev. | Min | Pctl(25) | Pctl(75) | Max |
|---------------------|---------|--------|--------|----------|-----|----------|----------|-----|
| days.after.earnings | 282,806 | -1.580 | 1      | 7.790    | -30 | -2       | 1        | 30  |

### I.3 Contrarian Effects Across Different Horizons

Results from Column 2 and 3 of Table 4 also raise the question of which horizon of past returns matter most to analyst future return expectations. To investigate this question, I follow [Greenwood and Shleifer \(2014\)](#) to estimate the non-linear regression of the form

$$\mu_{m,t}^A = a + b \sum_{j=0}^k \omega_j R_{m,t-2,t-j} + e_t \quad (43)$$

where  $\omega_j = \frac{\lambda^j}{\sum_{i=0}^k \lambda^i}$  is the weight on past returns and  $\lambda$  measures how quickly past return die out in analysts' memories. A value of  $\lambda$  equal to 1 implies the returns of different horizons are equally important in influencing analyst future expectation while a value smaller than one means more recent past returns are more important than distant ones.

For the empirical implementation, I run the regression (43) using monthly data based on cumulative quarterly returns that range from one quarter (returns that are lagged by three to six months) to 12 quarters, so 16 regressors in total. I correct for the auto-correlations in the return expectations by using Newey-West standard errors with 12 month lags. The non-linear least squares estimates are presented in Table 19.

The estimate of  $\lambda$  is 0.904, which shows that analysts pay more attention to recent past returns. Compared to the result found in [Greenwood and Shleifer \(2014\)](#), who estimate the average value of  $\lambda$  to be 0.56 based on a host of other subjective expectations, distant returns have much more important impact for analysts. This estimate can also be contrasted with [Mal-](#)

[mendier and Nagel \(2011\)](#), who find that *distant* but salient past history plays a role in investor market participation decisions.

Table 19: Analyst Return Expectations and Past Returns of Different Horizons

Non-linear least squares monthly time-series regressions of analyst return expectations of market returns for the next year on cumulative quarterly returns that range from one quarter (returns that are lagged by three to six months) to 12 quarters:

$$\mu_{m,t}^A = a + b \sum_{j=0}^k \omega_j R_{m,t-2,t-j} + e_t \quad (44)$$

where  $\omega_j = \frac{\lambda^j}{\sum_{i=0}^k \lambda^i}$ . Newey-West standard errors with twelve-month lags are shown in brackets. Sample starts from 2002-01-01 and ends in 2018-12-31, a total of 204 months.

|            | a       | b       | $\lambda$ |
|------------|---------|---------|-----------|
| Estimate   | 0.152   | -0.845  | 0.866     |
| Std. Error | (0.009) | (0.372) | (0.064)   |

#### I.4 Constructing Index-level Price-Dividend Ratio

I follow [Adam et al. \(2017\)](#) to construct index-level price-dividend ratios on monthly frequency. Monthly data on the level of the S&P 500 index is denoted as  $P_t^{SP}$ , as well as the monthly holding returns on the index without dividend, or  $R_t^{ND}$  are from CRSP. The monthly total returns on S&P 500 index including dividend, or  $R_t^D$  are from Global Insight.<sup>44</sup>

The monthly total dividend is

$$D_t = \left( \frac{1 + R_t^D}{1 + R_t^{ND}} - 1 \right) P_t^{SP}$$

<sup>44</sup>CRSP computes itself a value-weighted total returns including dividends and without dividend. However, upon examine the monthly implied dividend series, I found outliers. For example the November 2014 monthly dividend is almost three times the magnitude than that of the dividend in any other months in 2013 or 2014. Therefore, I used the global insights total returns. The implied dividend series does not have the irregular pattern throughout the entire sample from 1970 to 2019.

and the annual dividend is the sum of total dividend in the last 12 months

$$D_t^A = \sum_{i=0}^{11} D_{t-i}$$

and the log price-dividend ratio used in this study is

$$pd_t = \log(PD_t) = \log\left(\frac{P_t}{D_t^A}\right)$$

Notice, since the return expectations from analysts are based on analysts' forecasts of *Price* of the stock instead of returns, the index return expectations by analysts correspond to

$$E_{t-1}^A(R_t^{ND}) = E_t^A\left(\frac{P_{t+1}^{SP}}{P_t^{SP}} - 1\right)$$

where the superscript on the expectation operator denote the analyst expectation.

## I.5 Merging Analyst Price Target Forecasts with EPS Forecasts

I construct an analyst-level historical coverage data set based on detailed analyst EPS and price target forecast data. Before each date an analyst issues a price target, I trace all of the EPS and price target they have ever issued in the past. This set of firms are defined as their coverage.<sup>45</sup>

The EPS forecast is the longest available analyst survey (dating back to 1980-01-01) and also has the best coverage. The IBES database identifies an analyst through a unique "analyst code," which I use to merge the price target file and the EPS forecast file.

I first create an EPS-based coverage list in which all the firms for which an analyst has ever issued an EPS forecast are included. The first-ever announced EPS estimate of an analyst is considered as the start of the analyst's career. Additionally, the first and the last (or current) date on which they issue an EPS estimate for a firm is recorded as the start/end of their coverage for that particular firm. A similar coverage list is created for the price target data set. Empirically, the price target coverage is a subset of the coverage of the EPS forecast for most of the analysts.

I make several filters to get rid of potential erroneous observations. First, I only include an

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<sup>45</sup>Admittedly, this coverage might not be complete. An analyst might be covering other firms and has not issued any EPS or price targets in the past for those firms. However, this potential under-estimation will not affect the results on the impact of past experience on the future price target forecasts, if the under-estimation is systematically correlated with the past experienced returns/earnings of the covered firms.

analyst's one- and two-fiscal year and one-fiscal quarter ahead forecasts when making the EPS coverage list. The reason is that these periods are the most commonly surveyed horizons and are less prone to errors. Second, if an analyst stops appearing in the EPS file and reappears after 36 months, I count the restarting date as the analyst's career start. This is because only very few observations (6% of all observations) actually do reappear after three years. Analysts do update the forecasts quite often. The reason for not updating is mostly because of an erroneous analyst identification code. Third, I delete analysts who cover more than 200 firms. The average number of firms covered by an analyst in the EPS data base is about 41 with a median of 33. Analysts who cover more than 200 firms are highly unusual, which amount to less than 0.1% of all observations.

Table 20: Summary statistics analyst-level historical coverage

Analyst-level historical coverage data set for analysts who issue at least one price target (*ptg*) or EPS estimate (*eps*) during the entire sample period from 1999-01-01 to 2020-02-01. Analyst-level detailed unadjusted EPS data set is from 1980-01-01 to 2020-02-02 and price target data set is from 1999-01-01 to 2020-02-01. *no.firms.covered* is the number of unique firms for which an analyst issues at least one price target and/or EPS forecasts. *no.firms.w.eps* and *no.firms.w.ptg* are the number of firms the analyst issued EPS forecasts or price targets, respectively, for. *no.firms.w.eps.only* and *no.firms.w.ptg.only* are the number of firms the analyst issued only an EPS forecast or price target, respectively, for. *no.firms.w.both* are the number of firms the analyst issued both an EPS forecast and price target for. *no.months.analyst.career* is the number of months from the first-ever price target or EPS to the last time the analyst issues a price target or EPS forecast.

| Statistic                | N      | Mean   | Median | St. Dev. | Min   | Pctl(25) | Pctl(75) | Max     |
|--------------------------|--------|--------|--------|----------|-------|----------|----------|---------|
| no.firms.covered         | 14,352 | 18.224 | 10     | 22.099   | 1     | 3        | 26       | 189     |
| no.firms.w.eps           | 14,352 | 17.160 | 9      | 21.782   | 0     | 2        | 24       | 186     |
| no.firms.w.ptg           | 14,352 | 12.663 | 6      | 15.745   | 1     | 2        | 18       | 168     |
| no.firms.w.eps.only      | 14,352 | 5.562  | 1      | 13.042   | 0     | 0        | 5        | 170     |
| no.firms.w.ptg.only      | 14,352 | 1.065  | 0      | 3.911    | 0     | 0        | 1        | 122     |
| no.firms.w.both          | 14,352 | 11.598 | 5      | 15.133   | 0     | 1        | 16       | 167     |
| no.months.analyst.career | 14,352 | 82.932 | 51.567 | 85.366   | 0.000 | 15.633   | 126.067  | 462.433 |

The EPS-based coverage list is merged with the analyst-level price target issuance data to obtain the coverage history for each individual analyst that *ever issues at least one price target*.

Table 21 summarizes the analyst-by-analyst coverage data set. The data set contains more than 14 thousand analysts. The average number of firms an analyst covers is about 18 firms, consistent with the industry standard of about 10 to 30 firms per analyst. The coverage is skewed to the left, with a median analyst only covering 10 firms.

Typically, an analyst submits more EPS forecasts than price targets for the firms they cover. Among the 18 firms that an average analyst covers, more than 16 have an EPS forecast and only one firm has only price targets but no EPS forecasts. The less price target forecasts as compared to EPS is consistent with the facts documented in previous literature, such as [Da et al. \(2016\)](#).

This data set contains the point-in-time data for all the firms for which an analyst has ever issued EPS or price target forecasts, before they issue a new price target. Furthermore, it also has the first-ever issuance the analyst ever makes for either EPS or price targets, which I use to construct the number of years experience variable. This data is the basis for studying the impact of experience on analysts' return expectations.

This heterogeneity in the duration of analysts' careers is helpful for the analysis on analysts' past experience. A median analyst typically lasts for about four years between their first and last issuance, with a standard deviation of about seven years. Notice that there are some veterans who go on to have a career spanning almost four decades. One such analyst that I am able to trace online is Chuck Cerankosky from Northcoast Research, who started his career at Rouston Research in 1979 and is still active. Notice that this sample only includes the analysts who have ever issued a price target. Analysts who only issue EPS estimates but not price targets have a median career span of about 31 months. The analysts who issue both EPS estimates and price targets in this sample have substantially longer professional careers.

Table 21: Summary Statistics: Analyst Coverage History

This table reports summary statistics for the number of forecasts, price targets, and firms covered by analysts. *Coverage.eps.forecasts.per.analyst* (*Coverage.price.targets.per.analyst*) is the number of unique firms for which an analyst issues at least one EPS (price target) forecast in a given calendar year; *Overlap.firms.per.analyst* is the number of firms for which an analyst issues at least one EPS forecast and one price target forecast in a given calendar year. *No.firms.covered.EPS.analyst.career* (*No.firms.covered.PTG.analyst.career*) is the unique number of firms for which an analyst has issued at least one EPS (price target) forecast over their career; *Total.No.months.analyst.career* is the total number of months between the first and the last time an analyst ever issues an EPS/Price Target forecast in the data base.

(a) Point-in-time coverage statistics

| Statistic                          | N      | Mean   | Median | St. Dev. | Min | Pctl(25) | Pctl(75) | Max |
|------------------------------------|--------|--------|--------|----------|-----|----------|----------|-----|
| Coverage.eps.forecasts.per.analyst | 55,093 | 13.084 | 12     | 9.407    | 1   | 5        | 19       | 116 |
| Coverage.price.targets.per.analyst | 55,093 | 9.110  | 7      | 7.496    | 1   | 3        | 14       | 81  |
| Overlap.coverage.per.analyst       | 55,093 | 8.294  | 6      | 7.142    | 1   | 2        | 13       | 73  |

(b) No. firms covered over an analyst's career

| Statistic                           | N      | Mean   | Median | St. Dev. | Min   | Pctl(25) | Pctl(75) | Max     |
|-------------------------------------|--------|--------|--------|----------|-------|----------|----------|---------|
| No.firms.covered.EPS.analyst.career | 12,058 | 18.639 | 11     | 21.625   | 1     | 3        | 26       | 184     |
| No.firms.covered.PTG.analyst.career | 12,058 | 13.048 | 7      | 15.062   | 1     | 2        | 19       | 127     |
| Total.No.months.analyst.career      | 12,058 | 75.905 | 47.417 | 78.798   | 0.000 | 17.500   | 109.933  | 459.267 |

The analyst-level point-in-time coverage data is then merged with both the daily stock returns from the CRSP and the quarterly firm-level earnings data to obtain the analyst-level experience returns and earnings variables used in the analysis.

## I.6 A Simplified System With Dividend Expectations

$$\Delta \hat{d}_{t+1} = \frac{1}{L(\phi_g)} \hat{g}_{t+1}^A + \epsilon_{\Delta d, t+1} \quad (45)$$

$$\hat{d}p_{t+1} = \beta \hat{d}p_t + \frac{B_{dp,g}(\beta, \phi_g)}{L(\phi_g)} \hat{g}_{t+1}^A + \kappa_\mu \epsilon_{\mu, t+1} - \kappa_g \epsilon_{g, t+1} \quad (46)$$

$$\hat{g}_{t+1}^A = \phi_g \hat{g}_t^A + L(\phi_g) \epsilon_{g, t+1} \quad (47)$$

where

$$\begin{aligned}
L(\phi_g) &= \sum_{k=0}^3 \phi_g^k \\
\kappa_\mu(\beta) &= \frac{1}{1 - \rho\beta} \\
\kappa_g(\phi_g) &= \frac{1}{1 - \rho\phi_g} \\
B_{dp,\mu}(\beta, \phi_g) &= \kappa_g(\beta - \phi_g)
\end{aligned}$$

and

$$\begin{pmatrix} \epsilon_{\Delta d, t+1} \\ \epsilon_{\mu, t+1} \\ \epsilon_{g, t+1} \end{pmatrix} \sim N \left[ 0, \begin{pmatrix} \sigma_d^2 & \sigma_{\mu d} & \sigma_{gd} \\ \sigma_{\mu d} & \sigma_\mu^2 & \sigma_{\mu g} \\ \sigma_{gd} & \sigma_{\mu g} & \sigma_g^2 \end{pmatrix} \right] \quad (48)$$

This system from Equation (45) to (48) is a linear system of underlying shocks and non-linear with respect to parameters

$$\theta = (\phi_g, \beta, \sigma_d, \sigma_\mu, \sigma_g, \sigma_{\mu d}, \sigma_{gd}, \sigma_{\mu g})'$$

Given the normality assumption in (48), the system can be estimated by maximum-likelihood estimation. I write the system into state-space form.

The observable vector is

$$y_{t+1} = \begin{pmatrix} \Delta \hat{d}_{t+1} \\ \hat{dp}_{t+1} \\ \hat{g}_{t+1}^A \end{pmatrix}$$

and the latent processes

$$\alpha_t = \begin{pmatrix} \hat{g}_{t+1}^A \\ \hat{g}_t^A \\ \hat{dp}_{t+1} \\ \hat{dp}_t \\ \epsilon_{d, t+1} \\ \epsilon_{\mu, t+1} \\ \epsilon_{g, t+1} \end{pmatrix}$$

The dynamics of the measurement equations are captured by

$$\begin{pmatrix} \Delta \hat{d}_{t+1} \\ \hat{d}p_{t+1} \\ \hat{g}_{t+1}^A \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{L(\phi_g)} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{g}_{t+1}^A \\ \hat{g}_t^A \\ \hat{d}p_{t+1} \\ \hat{d}p_t \\ \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 0 & \frac{1}{L(\phi_g)} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$d = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\eta_t = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$H = 0_{3 \times 3}$$

and the state-equation is characterized by

$$\begin{pmatrix} \hat{g}_{t+1}^A \\ \hat{g}_t^A \\ \hat{d}p_{t+1} \\ \hat{d}p_t \\ \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \phi_g & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\kappa_g(\beta - \phi_g)}{L(\phi_g)} & 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{g}_t^A \\ \hat{g}_{t-1}^A \\ \hat{d}p_t \\ \hat{d}p_{t-1} \\ \epsilon_{d,t} \\ \epsilon_{\mu,t} \\ \epsilon_{g,t} \end{pmatrix} + \begin{pmatrix} 0 & 0 & L(\phi_g) \\ 0 & 0 & 0 \\ 0 & \kappa_\mu(\beta) & -\kappa_g(\phi_g) \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix}$$

$$T = \begin{pmatrix} \phi_g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\kappa_g(\beta - \phi_g)}{L(\phi_g)} & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$c = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$\epsilon_t = \begin{pmatrix} \epsilon_{d,t+1} \\ \epsilon_{\mu,t+1} \\ \epsilon_{g,t+1} \end{pmatrix}$$

$$R = \begin{pmatrix} 0 & 0 & L(\phi_g) \\ 0 & 0 & 0 \\ 0 & \kappa_\mu(\beta) & -\kappa_g(\phi_g) \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q = \begin{pmatrix} \sigma_d^2 & \sigma_{\mu d} & \sigma_{gd} \\ \sigma_{\mu d} & \sigma_\mu^2 & \sigma_{\mu g} \\ \sigma_{gd} & \sigma_{\mu g} & \sigma_g^2 \end{pmatrix}$$

where  $\Sigma_x$  has been pre-estimated using the OLS residuals. The system has an initial state vector

$$\alpha_{1|0} = (I_7 - T)^{-1}c$$

$$vec(P_{1|0}) = (I_{49} - T \otimes T)^{-1}vec(RQR')$$